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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

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MAY 1924

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THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
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VOLUME LIX MAY 1924 NUMBER 4

PRESSURES AT THE SUN'S SURFACE[1]

By HENRY NORRIS RUSSELL and JOHN QUINCY STEWART

ABSTRACT

Estimation of solar pressures.—The available methods of estimating the gas pressures which prevail at the sun's surface are based upon: (1) Pressure shifts of Fraunhofer lines. (2) Sharpness of lines sensitive to pressure. (3) General opacity of the outer regions of the sun. (4) The absence of scattered sunlight in the flash spectrum. (5) Widths of Fraunhofer lines. (6) The gravitational equilibrium of the outer regions of the sun. (7) Radiation pressure. (8) Ionization and chemical equilibria in the solar atmosphere. Previous work is reviewed, and new material presented under (2), (4), (5), and (8). All lines of evidence agree with the conclusion that the total pressure of the *photospheric gases* is less than 0.01 *atmosphere*, and that the average pressure in the *reversing layer* is not greater than 0.0001 atmosphere.

Constitution of the solar atmosphere.—It is probable that the *chromosphere* is supported by radiation pressure, and that the gas pressure at its base is of the order 10^{-7} atm. In the next 200 km (the *reversing layer*) the pressure increases to 10^{-2} atm. Below this level, general absorption produces rapidly increasing opacity, resulting in the *photosphere*. The whole amount of matter above the photosphere is of the order 10^{-4} gm/cm^2, equivalent to a layer of ordinary air 10 feet thick.

Similar considerations apply to the great majority of *stellar atmospheres*. Thus it is probable that the lines in most stellar spectra are produced substantially under vacuum conditions.

Widening of hydrogen lines by Stark effect.—A more detailed calculation shows that the extreme width of Hβ in a stellar spectrum, due to Stark effect in the ionized gas, may be more than one hundred times the mean width recently calculated by Hulburt under certain simplifying assumptions. This recalculation leads to much lower pressures than Hulburt computed.

Dissociation of TiO_2 vapor in the solar atmosphere.—Atkinson has calculated this, assuming the oxygen to be in the molecular state. It is almost certainly in the atomic. This modifies his equations, and leads to higher computed pressures.

In each of the foregoing cases the recalculation leads to values of the pressure in agreement with the results of other methods.

[1] Read before the National Academy of Sciences, April 24, 1923.

The problem of determining pressures in the solar atmosphere and photosphere is difficult, and a complete solution must await further progress in the physics of gases at high temperatures. Several lines of attack are already open; and these lead to very low values, indicating that the total pressure of the photospheric gases is probably less than 0.01 atmosphere, and that the pressure in the reversing layer is not much greater than 0.0001 atmosphere.

1. THE PRESSURE-SHIFTS OF FRAUNHOFER LINES

It was formerly believed that the pressure-shifts of Fraunhofer lines indicated a pressure of from one to two atmospheres in the reversing layer.[1] Later investigations, however, have shown that the pressure must be much lower. So many other causes can alter both laboratory and solar wave-lengths that the disentanglement of the pressure-shift is very difficult. Dr. St. John authorizes the statement that unpublished investigations at Mount Wilson, using lines as free as possible from disturbing influences, indicate that the pressure in the reversing layer is but a small fraction of an atmosphere—with a probable error about equal to the estimated pressure itself.

2. THE SHARPNESS OF LINES SENSITIVE TO PRESSURE

J. Evershed[2] has pointed out that the existence of distinct hydrogen lines in the spectrum of any star shows that the pressure in the reversing layer cannot exceed half an atmosphere. A more delicate test is afforded by certain lines of such elements as chromium (λλ 4111, 4097, 3912),[3] barium (λλ 3421, 3183, etc.),[4] and calcium (λλ 4355, 4108, 3972),[5] which are very diffuse when produced in the laboratory at atmospheric pressure and become sharp only under vacuum conditions. They are present as fine sharp lines in the solar spectrum, which indicates a decidedly low pressure, probably much less than 0.1 atmosphere, though numerical estimates are difficult.

[1] C. G. Abbot, *The Sun*, p. 98, 1911.

[2] *Monthly Notices*, **82**, 394, 1922.

[3] A. S. King, *Astrophysical Journal*, **41**, 110, 1915.

[4] *Ibid.*, **48**, 32, 1918.

[5] F. A. Saunders (personal letter).

3. THE GENERAL OPACITY OF THE OUTER REGIONS OF THE SUN

Direct observations show that the solar photosphere is effectively opaque in thickness of the order of 100 km (the size of the finest visible detail), and that the chromosphere and reversing layer are effectively transparent up to 10,000 km. The theory of the opacity of gases at high temperatures permits estimates of the pressure in both regions. Light passing through the outer portions of the sun is attenuated by at least three processes: (*a*) scattering of radiation by free electrons in the ionized gas; (*b*)thermal absorption of radiation by the free electrons; (*c*) scattering of radiation by the bound electrons.

Let F, with the appropriate subscript, represent the distance in kilometers required to reduce the intensity in the direct beam by the factor $1/e$ (where e is the "base of logarithms"). Then, owing to thermal absorption of radiation by free electrons,[1]

$$F_2=(64\times 10^{-5})(1+i)^2/ip^2,$$

(for wave-length 6×10^{-5} cm, and a temperature of 5000° absolute), where p is the total gas pressure in atmospheres and i the ratio of the number of free electrons to the number of atoms (ionized or not).

For scattering by free electrons

$$F_1=10(1+i)/ip$$

(for a temperature of 5000° absolute).

As for scattering by bound electrons, the well-known Rayleigh formula leads to

$$F_3=(3\times 10^{-5})n\lambda^4/32\,\pi^3(\mu-1)^2,$$

where λ is the wave-length in centimeters, μ the index of refraction for radiation of the given wave-length, and n the number of scattering centers (supposed alike) per cubic centimeter. It is readily shown that for $\lambda=6\times 10^{-5}$ and a temperature 5000° absolute,

$$F_3=2100/y^2p,$$

where y is the ratio of the value of $(\mu-1)$ for the gas to the value of $(\mu-1)$ for air at the same density. Presumably y is, in general,

[1] J. Q. Stewart, *Nature*, **111**, 186, 1923; *Physical Review*, **22**, 324, 1923.

somewhat greater than unity for the metallic vapors of the solar atmosphere, since the index of refraction is large in the neighborhood of absorption lines. For fixed values of i and p, F_1 and F_3 are proportional to the absolute temperature, T; and F_2 to $T^{\frac{1}{2}}$.

Classical electromagnetic theory readily shows that a bound electron scatters less energy than a free electron from radiation of frequency less than 0.71 times its natural frequency, ν_0, but more than twice as much from radiation in the frequency range 0.71 ν_0 to 1.85 ν_0. Therefore, when the ionization is appreciable, F_1 probably is less than F_3, except in the neighborhood of Fraunhofer lines.

Saha's theory permits calculation of ionization as a function of the temperature and the gas pressure.[1] Accordingly, F_1 and F_2 can be calculated. Table I gives such results for an illustrative solar atmosphere of sodium, at a temperature of 5000°.

TABLE I

p Pressure, in Atmospheres of Sodium Vapor and Free Electrons	i Fraction of Ionization	F_1 Kilometers Distance to Reduce Intensity to 1/*e*th	F_2 Kilometers Distance to Reduce Intensity to 1/*e*th
		Scattering	Absorption
1	0.062	170	0.012
10^{-1}	0.19	630	0.47
10^{-2}	0.53	2900	28
10^{-3}	0.89	21,000	2600
10^{-4}	0.99	200,000	260,000

A cursory examination of the tabulated values of F_2 indicates that photospheric opacity sets in at a depth in the sun where the pressure is 0.01 atmosphere or less. When observations are taken of the flash spectrum, the grazing line of sight runs for tens of thousands of kilometers through the solar chromosphere. Yet a continuous spectrum, such as would be produced by temperature radiation, is not observed in the flash. It is probably safe to estimate that the brightness of such a continuous spectrum is less than 1/100 that of the photosphere. Thus, if emission can be assumed proportional to absorption, the values of F_2 indicate that

[1] M. N. Saha, *Philosophical Magazine*, **40**, 472, 1920; H. N. Russell, *Astrophysical Journal*, **55**, 119, 1922.

the pressure in the chromosphere (or upper reversing layer) is less than 10^{-4} atmosphere.

4. THE ABSENCE OF SCATTERED SUNLIGHT IN THE FLASH SPECTRUM

The gas in the region where the flash spectrum originates is receiving solar radiation from a hemisphere and scattering it to the sphere. A layer of thickness d (provided d is small compared with F_1, and F_1 small compared with F_2) will, therefore, shine by diffused sunlight with a surface brightness of the order $d/2F_1$ times that of the photosphere. Now for a layer rising 300 km above the photosphere the average value of d is about 20,000 km. Since $d/2F_1$ may be estimated as less than 1/100, F_1 must be at least 10^6 km, and, by Table I, the pressure can hardly exceed 2×10^{-5} atmosphere.

5. THE WIDTH OF FRAUNHOFER LINES

A complete theory of the Fraunhofer lines, which shall quantitatively relate their intensities and widths to the amounts and densities of various substances present in the solar atmosphere, has never been formulated. The following three influences of abundance or density on line width, in a moderately rarefied gas, have been suggested.

a) Line width increases with the number of "absorbing" molecules in the line of sight, as described by classical electromagnetic theory, taking also into account the Doppler effect of thermal molecular velocity.

b) Molecular collisions broaden lines, in a manner which, with special assumptions, can be dealt with by electromagnetic theory.

c) Lines broaden with increasing density owing to the Stark effect on the absorbing molecules of fields due to neighboring ions and free electrons in an ionized gas.

The effects (*a*), of Doppler broadening and number of molecules, have been discussed as follows by Stewart,[1] for the case of a cool rarefied gas, such as the solar reversing layer, and employing the electromagnetic theory as outlined by Lorentz. In this case the dark lines may be supposed due to scattering of radiant energy rather than to

[1] *Astrophysical Journal*, **59**, 30, 1924.

true absorption. The calculated minimum number of molecules per square centimeter column along the line of sight required to produce an observable absorption line is

$$N' = (1.7 \times 10^{6}) \sqrt{T/M\lambda^{2}},$$

where T is the absolute centigrade temperature, M the molecular weight of the scattering molecules, and λ is in centimeters. The "observed width" of a dark line broader than, say, one-tenth angstrom—due to Rayleigh scattering close to resonance, where the molecular index of refraction is large—is calculated as

$$\Delta = (5.8 \times 10^{-13}) \lambda \sqrt{N},$$

where N is the number of molecules in a square centimeter column along the line of sight. These equations were arrived at with the aid of classical electromagnetic theory, and may on that account be open to objection: but it is hoped that the results are correct in order of magnitude.[1]

For the D lines of sodium in the solar spectrum Δ/λ is about 10^{-4}; and the indicated value of the number of sodium atoms per square centimeter column of the solar atmosphere effective in producing the D lines is 3×10^{16}. Thus, according to this theory, the partial pressure of non-ionized sodium in the reversing layer is very low. If the thickness of the reversing layer is 100 kilometers, the indicated partial pressure is of the order 10^{-9} atmosphere. Even this tiny amount of sodium is greater than can be supported against solar gravity by selective radiation pressure. Since most of the sodium is ionized, the partial pressure of all the sodium may be as great as 10^{-7} atmosphere.

The effect denoted (*b*) above has been treated by H. A. Lorentz.[2] He assumed that as the energy taken up by each molecule from radiation passing through the gas was lost (transformed into heat) at a collision, such "impact damping" would result in true absorption of radiant energy. This cause of opacity is negligible in comparison with the scattering described in (*a*) unless the gas

[1] The effects of absorption and temperature radiation in the reversing layer have been neglected.

[2] *Proceedings Royal Academy of Amsterdam*, **18**, 134, 1914.

is so dense that the number of collisions, r, per unit time per molecule is comparable with ac/λ^2, where c is the velocity of light and a is the radius of the electron calculated from the usual electromagnetic formula. If the area of cross-section of a molecule is πA^2, and if the average velocity of thermal agitation is u, then r is of the order $n\pi A^2 u$, where n is the total number of molecules per unit volume. Taking an average value of u in the solar atmosphere as 3×10^5, and of A as 3×10^{-8}, it is readily verified that the effect of impact-damping in broadening lines, in comparison with the effect of the "radiation-damping," which results in scattering, is important only at densities greater than 10^{-4} atmospheric—corresponding in the sun to pressures greater than 2×10^{-3} atmosphere.

The effect denoted (*c*) above was suggested by R. d'E. Atkinson[1] who roughly calculated it, with the aid of an elementary theory due to P. Debye.[2] Recently E. O. Hulburt[3] (who has very kindly permitted us to see his manuscript in advance of publication) has come, by application of a theory similar to that of Debye, to the conclusion that, if the broadening of the Balmer lines of hydrogen in certain stellar spectra is due to the Stark effect, the indicated pressures are of the order of several atmospheres. For the sake of simplicity he has neglected several factors, no one of much importance, but the combined influence of which is far from negligible.

Suppose that there are q charged corpuscles (ions and electrons) per unit volume of a gas, each of charge e. Then it may be shown from elementary considerations that the probability that no corpuscle shall be nearer a given molecule than the distance x is e^{-x^3/x_0^3}, where $\frac{4}{3}\pi q x_0^3 = 1$. Thus the probability that the nearest corpuscle shall lie between the distances x and $x+dx$ from a given molecule is $(3x^2/x_0^3)e^{-x^3/x_0^3}dx$.

Mechanical quadrature gives $2.67/x_0^2$ as the mean value of $1/x^2$; consequently $2.67e/x_0^2$, or $6.95\,eq^{\frac{1}{3}}$, is the mean field, regardless of sign, at a given point due to the nearest corpuscle of positive or negative charge e. It is evident that the mean total will be somewhat greater than this value; for the resultant field of the

[1] *Monthly Notices*, **82**, 396, 1922.

[2] *Phys. Zeits.*, **20**, 160, 1919.

[3] *Astrophysical Journal*, **59**, 177, 1924.

other corpuscles at the given point is in general smaller than that due to the nearest corpuscle; and as the relation between the respective directions of the two fields is random, the magnitude of the mean resultant is greater than that of either component. According to J. Stark,[1] the extreme components of Hβ are displaced 20 angstroms each way by a field of 1.04×10^5 volts. Assuming a linear relation, this amounts to a widening of 0.115 A per unit c.g.s.e.s. field. Thus if w is the mean width in angstroms of Hβ when the absorbing or emitting gas is ionized,

$$w=0.80\,eq^{\frac{1}{3}},$$

where $e=4.77\times10^{-10}$ e.s. units. Hulburt assumes the mean field to be $\frac{1}{3}eq^{\frac{1}{3}}$, and takes the widening as half the distance between extreme components. Hence his value for w is about one-fortieth of the foregoing.

Replacing q by $2(2.7\times10^{19})(273\ p'/T)$, where p' is the pressure of the free electrons in atmospheres, and T is the absolute temperature of the gas,

$$w=(2.3\times10^5)(p'/T)^{\frac{1}{3}}.$$

The observed width, w', in the case of a strong absorption line will be many times greater than the "mean" width, w; for in such a case a small fraction of the whole number of active molecules will suffice to produce considerable opacity, and those which lie in the strongest ionic fields will evidently be responsible for the edges of the line. For one case in one thousand, $x<0.1\ x_0$, and the field is accordingly greater than thirty-seven times its mean value.

For the sun, taking $T=5000°$ and $p'=10^{-4}$, the foregoing equation gives $w=1.7$ A. When $p'=10^{-5}$, $w=0.4$. The observed w' for Hβ is about 0.6. For α Lyrae, with $T=10{,}000°$, $p'=10^{-4}$, w is 1.1 A. The observed w' is about 20. In view of the strength of the lines in these two cases and of the probability that other causes of widening are effective, and in consideration also of the relation of the electron pressure to the total pressure, the conclusion is indicated that the latter is of the order 10^{-5} in the sun, and 10^{-4} in α Lyrae, in the region where the lines are produced.

[1] *Annalen der Physik*, **48**, 202, 1915.

6. THE GRAVITATIONAL EQUILIBRIUM OF THE OUTER REGIONS OF THE SUN

Eddington's theory of the internal constitution of the sun indicates that the pressure in the photosphere is considerably less than 0.1 atmosphere. The following relation between pressure and temperature is given:[1]

$$10^6 p = \beta a T^4/3(1-\beta),$$

where $10^6 p$ is the gas pressure in c.g.s. units (p being measured in atmospheres), and a is Stefan's constant, or 7.6×10^{-15}. The quantity β is the ratio of the gas pressure to the sum of the gas pressure and the radiation pressure. For the sun,[2] $\beta=0.971$. Accordingly $T^4=(1.2\times10^{19})p$. Thus at a depth in the sun where p is as great as 0.01 atmosphere, T is about 19,000°.

Although Eddington's theory does not hold as far out as the photosphere, it may be a fair approximation at a depth where the temperature is 19,000°. The density of the material in that underlying region can hardly be less than that in the photosphere. Thus, owing to the lower temperature alone, the photospheric pressure should be at least five times less than 0.01 atmosphere.

7. RADIATION PRESSURE

Saha[3] has suggested that selective radiation pressure may be a factor in supporting the solar atmosphere. Eddington[4] has pointed out that the whole observed radiation of the sun is capable of supporting a mass of only 8×10^{-5} gm/cm² against its gravity. The absorbed radiation (represented by the Fraunhofer lines) may be regarded as capable of holding up 10^{-5} gm/cm², which in a layer 400 km thick would give a mean density of 2.5×10^{-13} gm/cm³. This corresponds to a mean pressure of 10^{-7} atmosphere for atomic hydrogen at 5000°, and from 1/10 to 1/20 as much for metallic vapors.

As stated above, it has been shown that the quantity of material theoretically required to produce the broader Fraunhofer lines is

[1] A. S. Eddington, *Astrophysical Journal*, **48**, 205, 1918.

[2] A. S. Eddington, *Monthly Notices*, **83**, 104, 1922.

[3] *Astrophysical Journal*, **50**, 220, 1919; *Nature*, **107**, 488, 1921.

[4] *Monthly Notices*, **80**, 723, 1920.

much greater than selective radiation pressure can support against solar gravity. Thus it seems unlikely that such pressure is an important factor in the gravitational equilibrium of the reversing layer, although it may well be influential in the equilibrium of the chromosphere.

A. Pannekoek[1] has shown theoretically that the molecular weight of an ionized gas is much reduced by the division of weight between ions and electrons. Consequently electrostatic forces must be taken into consideration in the equilibrium of the solar atmosphere.

8. IONIZATION AND CHEMICAL EQUILIBRIA IN THE SOLAR ATMOSPHERE

The Nernst equation gives the degree of dissociation of a chemical compound (or ionization of an element) as a function of the temperature and pressure of the gas, so that the pressure may be found if the temperature and degree of dissociation can be estimated.

a) Ionization of calcium.—In the case of ionization, the Nernst equation may be written, following Saha,[2] as

$$\log \frac{x}{1-x} + \log p_e = -\frac{5036\, I}{T} + 2.5 \log T - 6.5 ,$$

where x is the fraction of atoms, of ionization potential I volts, which are actually ionized, T the absolute temperature, p_e the partial pressure of the free electrons (in atmospheres), and the logarithms are to the base 10.

Now the ultimate lines of ionized calcium (H and K) are much stronger than those of neutral calcium (λ 4226) even in the spot spectrum. The sun-spot spectrum requires about six times as long an exposure as that of the photosphere at λ 6000° and ten times as long at λ 4000°. Employing these data and taking the effective temperature of the photosphere as 6000°, that of the spot is calculated as 4200°. The temperature in the reversing layer above a spot should be about 0.85 of this, and may be taken as 3600°. For calcium ($I = 6.09$ volts) we have then $p_e = 8 \times 10^{-7}\ (1-x)/x$. The last factor is the ratio of the number of neutral atoms to that of ions, and must be less than unity since the lines of $Ca+$ are

[1] *Bull. Astro. Inst. Netherlands*, 1922, No. 19.

[2] *Philosophical Magazine*, **40**, 472, 1920.

stronger than those of *Ca*, even in spots. Hence p_e is of the order of 10^{-7}. It is probable that the ionization of calcium is considerably aided by the absorption of photospheric radiation, which puts the atoms into "excited" states in which they are easier to ionize. Allowance for this would increase the permissible value of p_e but, even so, it would be of the order 10^{-6}. Since several other abundant elements (*Na*, *K*, *Al*,) are as easily ionized as *Ca*, the total pressure is probably not more than $10\ p_e$.

b) *Dissociation of titanium dioxide.*—This has been studied by Atkinson,[1] assuming the reaction $TiO_2 \rightleftarrows Ti+O_2$. At solar temperatures and pressures, however, oxygen is probably completely dissociated, making the reaction $TiO_2 \rightleftarrows Ti+2O$. This should increase the heat of reaction by the heat of dissociation of O_2, which may be estimated as 70,000 calories,[2] making $U = 285{,}000$ cal. Taking C_p for atomic oxygen to be 5.0, and its chemical constant as +0.2 (from the Sackur-Tetrode relation), and the other data from Atkinson, the dissociation equation becomes

$$\log \frac{x^3P^3}{(1-x)(1+2x)^2} = -\frac{62300}{T} + 3.75 \log T - (2.7\times 10^{-4})T - 2.0\,,$$

where P is the combined pressure of TiO_2 and its constituent elements, and x the fraction dissociated. Setting $T = 3600°$ and remembering that x is very nearly unity (since the band spectrum of the oxide is at the point of disappearing), we find

$$P^2 = 10^{-8}(1-x)\,,$$

indicating that P lies between 10^{-5} and 10^{-6} atmospheres. The whole atmospheric pressure may be of the order of $100\ P$ (judging from the fact that TiO_2 forms very nearly 1 per cent of the earth's crust).

The foregoing equation holds good if titanium and oxygen are present in chemically equivalent proportions. If not, the order of magnitude of the result is unaffected, as Atkinson has pointed out.

Since this paper was read, but before its publication, R. H. Fowler and E. A. Milne,[3] from consideration of the conditions under which the intensity of arc lines of subordinate series, or of enhanced lines

[1] *Op. cit.*

[2] I. Langmuir (personal communication).

[3] *Monthly Notices*, **83**, 403, 1923.

of any sort, should be a maximum, have derived very similar values for the pressure in reversing layers.

THE CONSTITUTION OF THE SOLAR ATMOSPHERE

The results of these numerous methods of investigation are all consistent with the following conception of the solar atmosphere. At the top is a deep layer, the *chromosphere*, in which the gases are held up by radiation pressure, acting on individual atoms. The pressure and density in this layer increase slowly downward (as gravity somewhat overbalances radiation pressure), and the pressure at its base may be of the order of 10^{-7} atmospheres, or 0.0001 mm of mercury. Below this level, gravity is predominant in the equilibrium, and the pressure increases rapidly with depth—the temperature remaining nearly constant, and not far from 5000°, so long as the gases are transparent. This region is the *reversing layer*. When the pressure reaches 0.01 atmosphere, the general absorption begins to render the gas hazy. This opacity increases greatly with the pressure, and the reversing layer passes, by a fairly rapid transition, into the *photosphere*, which on the scale in which we have to study it resembles an opaque mass. As soon as the opacity becomes important the temperature rises in accordance with the theory of radiative equilibrium as developed by Schwarzschild and Eddington. The observed effective photospheric temperature is a mean value for the layers from which radiation escapes to us.

In the reversing layer, with a temperature near 5000°, and a mean molecular weight probably of about 20, the pressure and density must change tenfold for a change in depth of approximately 40 km. The assumption of an increase of density by the factor 10^5 between the chromosphere and photosphere leads to a depth of 200 km for the layer. A rough calculation from Table I indicates that fully 80 per cent of the light originating at this depth should escape from the sun, and practically none of that from 50 km lower. The transition to the photosphere is therefore rapid enough to give the observed sharp limb of the sun in accordance with Abbot's explanation,[1] and yet gradual enough to account for the change in

[1] *Op. cit.*, p. 241.

brightness toward the limb, as shown in detail by B. Lindblad[1] and Milne.[2]

Spectroscopically, the upper chromosphere should be responsible for the bright lines visible at the limb with small instruments, and for the dark central cores of the hydrogen and calcium lines in the ordinary spectrum. The flash spectrum probably arises both from the lower chromosphere and upper reversing layer. The stronger Fraunhofer lines should be absorbed mainly in the upper part of the reversing layer—the lower and denser part accounting for the wings of dense lines, for the fainter lines (whether due to rare elements or to statistically improbable quantum-changes in atoms of abundant elements), and for the band spectra, both in spots and on the disk. All these deductions appear to be in excellent agreement with observations, and most of them are not new.

The whole amount of matter above the photosphere (sufficient to produce a pressure of 0.01 atmosphere) is 0.4 gm/cm^2—equivalent to a layer of ordinary air 10 feet thick. This seems surprisingly small, but consideration of the absorption produced by much thinner layers of highly rarefied vapor in the laboratory shows it to be abundantly sufficient to account for the Fraunhofer lines. (Cf. Stewart's theoretical discussion.)[3]

The considerations here developed are evidently applicable to the stars at large (as has been suggested by Evershed and Atkinson.) In all stars as hot as the sun or hotter, and for the cooler giant stars, the atmospheres must be highly ionized and the pressure in the photosphere of the order of 10^{-2} atmospheres or lower. In the brightest giants (or c-stars) where the surface gravity must be very low, and the chromosphere and reversing layer correspondingly deep, effective opacity will be reached at a still lower pressure. The opposite conditions prevail in the fainter and cooler dwarf stars, in which the density at the photosphere may be considerable. The fact that the spectral lines are unusually sharp in c-stars and very diffuse in faint dwarfs is thus immediately explicable.

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[1] *Transactions of Royal Society of Science of Upsala*, **6**, 1, 1923.

[2] *Philosophical Transactions*, **223**, 201, 1922.

[3] *Op. cit.*

THE EFFECTIVE WAVE-LENGTH IN WHITE-LIGHT INTERFEROMETRY

By FRED ALLISON

ABSTRACT

A mathematical theory of the form of the interference fringes from a single source is developed. The theory is then illustrated by some computed instances. It is shown how this theory can be combined with experiment for the purpose of obtaining the effective wave-length for any double star of known black-body temperature. Good agreement is found between the values obtained by the present method and Anderson's experimental value for sunlight.

Practically all measurements with an interferometer involve a knowledge of the wave-length of the source. When white light is used, we are concerned with a certain mean value known as the "effective" wave-length. To reach an exact definition, imagine a perfectly monochromatic source, which at any time may replace the actual source, under such ideal control of the observer that at will he may cause to be emitted any wave-length of the visible spectrum. Suppose that one has under observation some particular phenomenon from a complex source, the first minimum of visibility, for instance, when the interferometer is set on a double star. We now imagine that the observer replaces the double star with the ideal monochromator, and runs the range of wave-lengths until he most nearly matches the phenomenon actually observed from the double star, without changing in any way the adjustment of the instrument. The wave-length thus found is defined as the effective wave-length. If the setting were made on the second minimum of visibility, however, the ideal monochromator would be required to give off a wave-length of different value. Similarly, the effective wave-length for the same source will vary, depending upon the particular phenomenon with which it is associated, as will appear from the more rigorous treatment to follow. The problem, then, is to devise some method for determining the effective wave-length associated with any set of phenomena, the ideal arrangement indicated above, of course, not being at our disposal.

Anderson[1] in a recent paper sets forth the need of the knowledge of the effective wave-length to an accuracy within a few tenths of 1 per cent. He outlines, upon a theoretical basis, a method which was first suggested to him in a communication from Professor C. M. Sparrow, whereby the effective wave-length of any star may be computed, and reports his laboratory determinations of the effective wave-length of sunlight. The present paper presents the results of certain computations and experiments which afford a test of the theory, and indicates some of the precautions that are necessary for work of the highest precision.

Anderson gives an account of earlier contributions to the subject, which need not be repeated. Reference should be made to a former paper of Anderson's,[2] which bears upon the same subject.

In the two papers just mentioned, Anderson has discussed the use of the interferometer in astronomical investigations. A brief recapitulation will help to make more concrete the theory which follows. If the telescope with the interferometer attachment is directed on a double star, the angular separation of whose components is α, the distance $2b$ between the centers of the two apertures of the interferometer is so adjusted that the first minimum of visibility of the fringes is observed. Such an appearance of the fringes is conditioned upon the bright fringes of one component falling upon the dark fringes of the other, the two systems of fringes being out of step by one half-fringe. The condition is realized when

$$\alpha = \frac{\lambda}{4b}. \tag{1}$$

If the source is a star of uniformly luminous circular disk of angular diameter β, the following relation holds for the first minimum of visibility of the fringes

$$\beta = \frac{1.22\,\lambda}{2b}. \tag{2}$$

The result of any interferometer measure (the position angle of a double star excepted) will be expressed in the form

$$\theta = K\lambda, \tag{3}$$

[1] *Astrophysical Journal*, **55**, 48, 1922. [2] *Ibid.*, **51**, 225, 1920.

where K is determined by the actual observation, and λ depends upon the object observed.

The expressions (1), (2), and (3) are readily derived upon the assumption of a monochromatic source, λ being the wave-length of the source. In the case of an actual star, the λ in each of the three equations is the effective wave-length associated with the phenomenon observed.

GENERAL THEORY

Suppose that we have a distant monochromatic point source from which light is allowed to come through a single aperture so as to produce a diffraction pattern on a distant screen parallel to the plane of the aperture. For our purpose only rectangular and circular apertures need be considered.

For a single rectangular aperture, the resultant amplitude at any point on the screen may be expressed by

$$R_1 = R_0 \frac{\sin(\phi/2)}{\phi/2}, \tag{4}$$

in which ϕ and R_0 represent, respectively, the range of phase and the maximum amplitude.

In the case of a single circular aperture, we may write for the resultant amplitude produced on the screen

$$R_1 = R_0 \frac{J_1(\phi/2)}{\phi/2}, \tag{5}$$

where $J_1(\phi/2)$ is Bessel's function of order unity.

The amplitude due to two similar apertures takes the form

$$R = 2R_1 \cos \frac{\phi b}{2a}, \tag{6}$$

in which a represents half the width of the aperture; b, half the distance separating the two apertures measured along the line joining their centers; and R_1, the amplitude for a single aperture.

For rectangular apertures (6) becomes

$$R = 2R_0 \frac{\sin(\phi/2)}{\phi/2} \cos \frac{\phi b}{2a}. \tag{7}$$

Squaring, we get for the intensity

$$I \equiv I_0 \frac{\sin^2(Aa)}{A^2a^2} \cos^2(Ab), \tag{8}$$

after putting $Aa = \phi/2$.

Similarly, the intensity for two circular apertures is given by

$$I \equiv I_0 \frac{J_1^2(Aa)}{A^2a^2} \cos^2(Ab). \tag{9}$$

Equations (8) and (9) may both be represented by the more general expression

$$I \equiv I_0 F(Aa) \cos^2(Ab), \tag{10}$$

F depending upon the form of aperture.

It remains to consider the extension of the source. The cases of interest are: (*a*) a two-point source, such as a double star; (*b*) an extended source, such as a stellar disk.

For a double-point source having angular separation of 2α and of unequal intensities proportional to I_1 and I_2, and for rectangular apertures, we may write (for small angles) for the intensity corresponding to any angle of diffraction θ, according to equation (10),

$$I(\theta) \equiv I_1 F\left[\frac{2\pi a}{\lambda}(\theta-\alpha)\right] \cos^2 \frac{2\pi b}{\lambda}(\theta-\alpha) + I_2 F\left[\frac{2\pi a}{\lambda}(\theta+\alpha)\right] \cos^2 \frac{2\pi b}{\lambda}(\theta+\alpha). \tag{11}$$

For a uniformly illuminated disk source of angular diameter 2α, the intensity as a function of the angle of incidence i is proportional to $\sqrt{1-\frac{i^2}{\alpha^2}}$, so that equation (10) becomes

$$I(\theta) \equiv I_0 \int_{-\alpha}^{\alpha} \frac{2\pi a}{\lambda}(i+\theta) \cos^2 \frac{2\pi b}{\lambda}(i+\theta) \sqrt{1-\frac{i^2}{\alpha^2}}\, di. \tag{12}$$

Expressions for other types of sources may similarly be formulated, the general form of the expressions being

$$I(\theta) \equiv I_0 \int\int F(i, \theta, \xi) di\, d\xi\,, \tag{13}$$

where ξ is the co-ordinate of an element of the aperture.

In the derivations thus far, we have assumed monochromatic sources. Accordingly, equation (13) expresses the intensity as a function of the co-ordinate of the element of apertures, the angle of incidence, and the angle of diffraction. In order to pass to the actual case of double stars and stellar disks, we must include in the equation a wave-length function. Equations (11), (12), and (13) will then be of the general form

$$I\left(\begin{matrix}\theta \\ \lambda_0\end{matrix}\right) \equiv I_0 \int\int\int F(\xi, i, \lambda, \theta) d\xi\, di\, d\lambda\,, \tag{14}$$

the integration with respect to λ being taken over the range of visible wave-lengths emitted by the source under consideration.

For a monochromatic source the equation corresponding to (14) is

$$I^{\theta}_{\lambda'} \equiv I_0 \int\int F(\xi, i, \lambda', \theta) d\xi\, di\,. \tag{15}$$

In the last two equations λ_0 and λ' denote, respectively, the effective wave-length and the monochromatic wave-length. These equations furnish a more rigorous definition of the effective wave-length. It is clearly a function of the slit-widths, the angle of incidence, and the angle of diffraction. Thus, it will not be the same for extended sources, such as stellar disks, as it is for two-point sources, such as the usual type of double star. We may now make the important statement that there is a definite meaning to be attached to the effective wave-length in so far as, for any particular phenomenon of visibility, equations (14) and (15) may be regarded as equivalent.

The problem now is, given the spectral distribution of the source, to find a basis for predicting the interference pattern. This really

amounts to carrying out the computations indicated in equation (14). In the absence of fuller knowledge, the source is taken as a black body, and it is assumed that the phenomena are observed visually by a normal eye through the atmosphere. It will appear from the sequel that these apparently rough assumptions are sufficient for attaining very good agreement with experiment. Three functions of the wave-lengths are thus involved. The spectral distribution E_λ is computed from Planck's formula. The atmospheric transmission A_λ was taken from data supplied by Dr. Anderson (and hence especially applicable to the conditions of his own experiments), and the visibility V_λ was taken from the A.I.S. visibility curve. The product $E_\lambda A_\lambda V_\lambda$ forms what may be called the significant wave-length function, as it gives the spectral distribution of the visual stimulus. The visual stimulus for the interval $d\lambda$ is given by

$$E_\lambda A_\lambda V_\lambda d\lambda \equiv \Phi(\lambda) d\lambda \,. \tag{16}$$

For two infinitely narrow slits and a source emitting light in the interval $d\lambda$ we may write for the visual stimulus at a point x of the screen

$$\Phi(x) \equiv \Phi(\lambda) \cos^2\left(\frac{\pi n x}{2\lambda}\right) d\lambda = \int_0^\infty \frac{c^2 h \lambda^{-5}}{e^{\frac{ch}{K\lambda T}} - 1} A_\lambda V_\lambda \cos^2\left(\frac{\pi n x}{2\lambda}\right) d\lambda \,, \tag{17}$$

where n merely determines our unit of length. n is here taken = 100.

The integral (17) was evaluated by quadratures. The upper and lower limits of integration were 7100 A and 4100 A, at each of which the integral is practically zero. The intervals were 100 A. The computations were carried out for values of x extending from 0 to 330, in intervals of 1 unit; and for the temperature 6800° K., the computations were continued to the value of $x = 780$. From an examination of these extreme values it was found unnecessary to go beyond the value $x = 330$ for the other temperatures. Four sets of computations were made for the temperatures 2000°, 4000°, 6800°, and 40,000° K.

In a similar manner, computations were next made for slits of finite width. This is a still more laborious undertaking because

of the more complicated expression to be integrated. The visual stimulus for finite slits may be put in the form

$$\Phi(x) \equiv \int_0^\infty \frac{c^2 h \lambda^{-5}}{e^{\frac{ch}{K\lambda T}} - 1} A_\lambda V_\lambda \frac{\sin^2\left[\frac{\pi a n x}{2b\lambda}\right]}{\left[\frac{\pi a n x}{2b\lambda}\right]^2} \cos^2\left[\frac{\pi n x}{2\lambda}\right] d\lambda . \qquad (18)$$

The integral was evaluated for slits having the ratios a/b equal to $\frac{1}{4}$ and $\frac{1}{2}$, both for temperature 6800° K.

Some results of interest furnished from the computations for both the infinitely narrow and also the finite apertures are collected in Table I. Certain results of the computations are also shown in Figure 1, *a* and *b*. In these plots the ordinates represent the visual stimulus multiplied by a constant so that for purposes of comparison they all have the same value at the origin, and the abscissae represent the distances on the screen measured from the center of the pattern. In Figure 1*a*, the solid curve is for zero slit-width; the curve represented by dots and dashes for finite slit-width, $a/b = \frac{1}{4}$; the dotted curve for finite slit-width, $a/b = \frac{1}{2}$. The curves of Figure 1*b* correspond to the temperatures 2000°, 4000°, and 40,000°, zero slit-width.

It will be noted from both Table I (p. 218) and the graphs that for infinitely narrow slits the fringes persist over a considerable range, while for the slits of finite width they are rapidly obliterated. No structure in the case of the ratio of 1 to 2 is recognizable beyond the second minimum. There is also to be observed, when the very narrow slits are employed, a progressive increase in the distance between successive maxima and minima, an effect to be expected from theoretical considerations. For the finite slits, however, a forward shifting along the *x*-axis of the minima and a backward shifting of the maxima were found to exist, a behavior pointed out by Anderson from analytical considerations, as also appears from equation (18). These characteristics of the interference pattern come into special consideration in the study of the pattern resulting from the superposition of two such fringe systems. The positions of maxima and minima recorded in Table I were taken by parabolic interpolation.

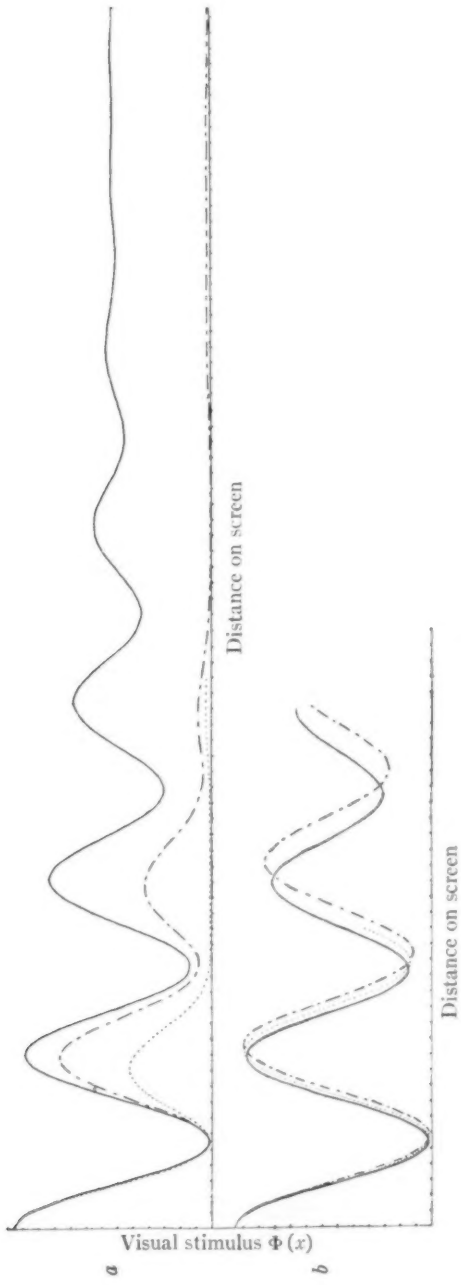


FIG. 1

a) Temperature, 6800°: ———, infinitely narrow slits; — · — ·, finite slits ($a/b = \frac{1}{4}$); . . . . . , finite slits ($a/b = \frac{1}{2}$)
b) Infinitely narrow slits: ———, $T = 40{,}000°$; . . . . . , $T = 4000°$; — · — ·, $T = 2000°$

TABLE I

	$T=2000°$ Slit-Width = 0			$T=4000°$ Slit-Width = 0			$T=6800°$ Slit-Width = 0			$T=40{,}000°$ Slit-Width = 0			$T=6800°$ Finite Slits $a/b=\frac{1}{4}$		
	x	Δ	$\frac{x-x_0}{n}$	x	Δ	$\frac{x-x_0}{n}$	x	Δ	$\frac{x-x_0}{n}$	x	Δ	$\frac{x-x_0}{n}$	x	Δ	$\frac{x-x_0}{n}$
First minimum	5785.6	5785.6	5785.6	5600.7	5600.7	5600.7	5518.8	5518.8	5518.8	5426.5	5426.5	5426.5	5521.6	5521.6	5521.6
First maximum	11593.6	5808.0	5796.8	11212.9	5612.2	5606.4	11051.7	5532.9	5525.8	10861.9	5435.4	5431.0	10848.0	5326.	5424.
Second minimum	17400.7	5807.1	5800.2	16842.1	5629.2	5614.0	16617.2	5565.5	5539.1	16363.4	5501.5	5454.5	16777.5	5930.	5592.
Second maximum	23235.8	5835.1	5809.0	.....	.....	.....	22209.4	5592.2	5552.4	21893.7	5530.3	5473.4	21750.	4972.	5438.
Third minimum	29098.7	5862.9	5819.7	.....	.....	.....	27833.2	5623.8	5566.6	27459.7	5566.0	5491.9	.....	.....	.....
Third maximum	.....	.....	.....	.....	.....	.....	33715.7	5882.5	.....	.....	.....	.....	.....	.....	.....
Fourth minimum	.....	.....	.....	.....	.....	.....	39100.0	.....	.....	.....	.....	.....	.....	.....	.....
	$T=6800°$ Finite Slits $a/b=\frac{1}{2}$														
First minimum	5534.	5534.	5534.	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
First maximum	10130.	4596.	5065.	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

x indicates position of maxima and minima.
Δ indicates distance between successive maxima and minima.
$\frac{x-x_0}{n}$ indicates distance from origin to respective maxima and minima divided by 1, 2, 3, ...

An interesting empirical relation is found to hold between the temperature and the distance from the origin to the first minimum, which we shall designate by λ_x. We may write

$$\lambda_x T + aT + b\lambda_x + c = 0\,, \tag{19}$$

a, b, and c being constants. By solving in the usual manner, using the temperatures 2000°, 6800°, and 40,000° and the corresponding values of λ_x for zero slit-width, we find for a, b, c the respective values -5407.6, 0, 7560. Substituting these values in (19), we get for $T = 4000°$ the value $\lambda_x = 5596.6$, as compared with 5600.7 from Table I. Equation (19) thus takes the simple form:

$$\lambda_x = \frac{7560}{T} + 5407.6\,, \tag{20}$$

an expression which will be seen of value for approximate effective wave-length determinations.

It was also of interest to find from the computed results given by equation (16) the wave-length of maximum luminosity for each of the temperatures. By parabolic interpolation values of the wave-length of maximum luminosity were calculated for the appropriate temperature, the values being 5834.6 A, 5623.8 A, 5524.3 A, 5424.1 A for the respective temperatures 2000°, 4000°, 6800°, 40,000°. The difference between these values, which are sometimes taken for the effective wave-length, and our values for the effective wave-length will be noted from Table II (p. 221). These relations are also shown in Figure 2 (p. 220), in which the visual stimulus of the different wave-lengths is plotted against the wave-length, the ordinates being multiplied by a constant so chosen as to give the same maximum ordinate for each temperature. It is interesting to note the decreasing shift of the maximum in passing from low to high temperatures.

The computations discussed thus far relate to a single point source. For a double star of equal and similar components, we have two fringe systems superposed. The resultant stimulus may be expressed by

$$\Phi\left(x+\frac{c}{2}\right)+\Phi\left(x-\frac{c}{2}\right)=\int_0^\infty \Phi(\lambda)\left[\cos^2\frac{\pi\left(x+\frac{c}{2}\right)n}{2\lambda}+\cos^2\frac{\pi\left(x-\frac{c}{2}\right)n}{2\lambda}\right]d\lambda\,, \tag{21}$$

where c is the relative displacement of the two fringe systems.

The first effort to establish a criterion for minimum visibility was to find by computation what relative shift in the two interference systems would yield a resultant pattern of minimum oscillation. The function $\Phi\left(x+\frac{c}{2}\right)+\Phi\left(x-\frac{c}{2}\right)$ was evaluated for c, equal, respectively, to 5400, 5500, 5600, 5700, and 5800 ($T=6800°$ and

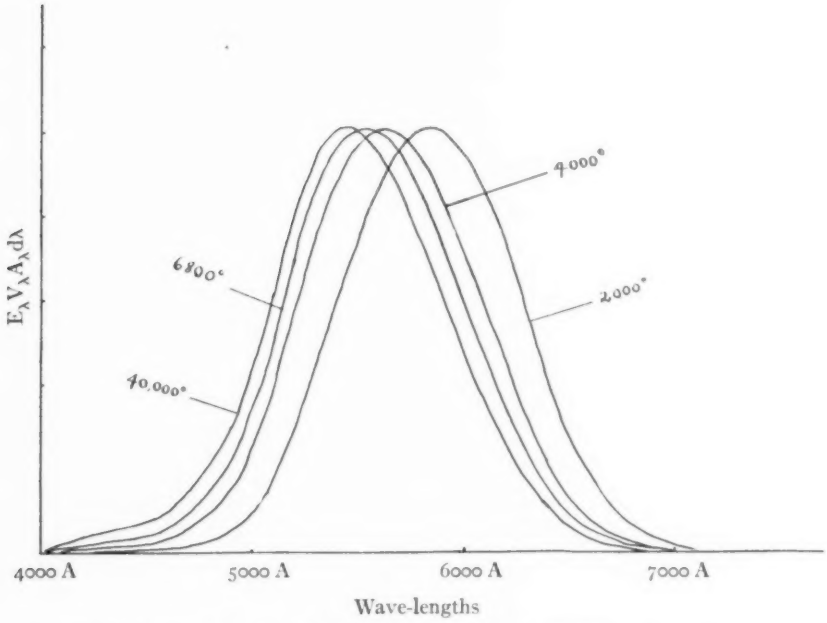


FIG. 2.—Curves showing distribution of luminosity for different temperatures

slit-width = 0). The results are shown graphically in Figure 3. Rayleigh[1] found that in a system of bright and dark bands the structure could be recognized with certainty when a difference of about 3 or 4 per cent existed between the luminosity of the darkest and brightest parts of adjacent bands. According to this, the contrast between the maxima and minima in Figure 3 (p. 222) is more than sufficient for the eye to detect structure, so that the fringes would never quite vanish. The value of c giving the first minimum oscilla-

[1] *Scientific Papers*, 3, 277.

tion was found by interpolation to correspond to an effective wave-length of 5611.3 A. It must be pointed out, however, as appears

TABLE II

EFFECTIVE WAVE-LENGTHS (VISUAL METHOD) INFINITELY NARROW SLITS

Observer	Temperature			
	2000°	4000°	6800°	40,000°
F. A.	5787.2 ±2.6	5610.4 ±2.1	5517.4 ±1.8	5423.2 ±2.4
	5790.7 ±4.1	5615.6 ±1.3	5519.4 ±2.8	5427.6 ±1.4
	5786.4 ±4.4	5608.8 ±1.8	5532.8 ±1.5	5430.9 ±0.7
	5790.4 ±4.4	5596.0 ±3.2	5528.6 ±3.4	5423.6 ±2.1
	5794.0 ±1.1	5605.6 ±3.8	5523.2 ±4.9	5419.4 ±2.1
Mean	5789.7 ±0.9	5607.3 ±2.2	5524.3 ±1.9	5424.9 ±1.3
J. B. B.	5781.8 ±1.6	5617.0 ±3.0	5526.8 ±2.1	5418.8 ±1.6
	5781.2 ±2.1	5616.8 ±1.8	5533.8 ±3.3	5423.9 ±2.7
	5779.4 ±1.4	5615.8 ±3.3	5538.4 ±4.6	5433.8 ±2.1
	5782.4 ±1.5	5610.6 ±1.9	5530.8 ±2.1	5436.2 ±1.6
	5779.6 ±3.9	5612.2 ±2.4	5492.8 ±2.8	5437.8 ±2.3
Mean	5780.9 ±0.4	5614.5 ±0.9	5524.5 ±5.5	5430.1 ±2.5
M. D.	5787.6 ±1.0	5618.8 ±1.5	5537.6 ±3.0	5437.6 ±2.1
	5779.4 ±2.8	5600.0 ±0.7	5505.4 ±5.5	5433.6 ±3.2
	5784.6 ±4.0	5603.4 ±2.9	5520.6 ±7.3	5391.6 ±3.1
	5779.4 ±2.6	5596.0 ±2.4	5528.4 ±2.2	5426.8 ±6.1
	5783.4 ±2.6	5603.2 ±3.1	5511.0 ±1.7	5440.4 ±4.5
Mean	5782.9 ±1.1	5604.3 ±0.7	5520.6 ±3.9	5426.0 ±6.0
L. G. H.	5792.4 ±2.4	5599.2 ±4.5	5537.2 ±1.1	5432.8 ±4.5
	5797.4 ±3.3	5606.2 ±4.1	5507.6 ±2.4	5426.6 ±2.4
	5787.2 ±1.8	5624.2 ±6.0	5532.0 ±3.9	5419.4 ±1.5
	5795.6 ±3.1	5605.8 ±3.6	5528.2 ±2.5	5409.8 ±2.2
	5807.8 ±3.9	5608.0 ±3.2	5528.2 ±3.5	5447.6 ±3.3
Mean	5796.1 ±2.3	5608.7 ±2.8	5526.6 ±3.4	5427.2 ±4.3
W. T. S.	5782.2 ±2.4	5622.4 ±4.3	5521.2 ±3.3	5427.8 ±1.3
	5786.0 ±4.6	5600.2 ±4.7	5520.2 ±1.8	5420.8 ±4.0
	5786.4 ±1.9	5612.8 ±2.2	5534.0 ±2.9	5423.2 ±2.4
	5798.4 ±2.0	5587.4 ±3.6	5510.6 ±3.1	5423.6 ±2.7
	5795.0 ±2.7	5599.8 ±2.6	5506.2 ±2.8	5416.4 ±3.6
Mean	5789.6 ±2.0	5604.5 ±4.2	5518.4 ±2.4	5422.4 ±1.3
Mean for all observers	5787.8 ±1.6	5607.8 ±1.2	5522.9 ±1.0	5426.1 ±0.6

in Figure 3, that this method is open to the criticism that the maximum ratio of adjacent maxima and minima corresponds to different

$T=6800°$ K. Infinitely narrow slits

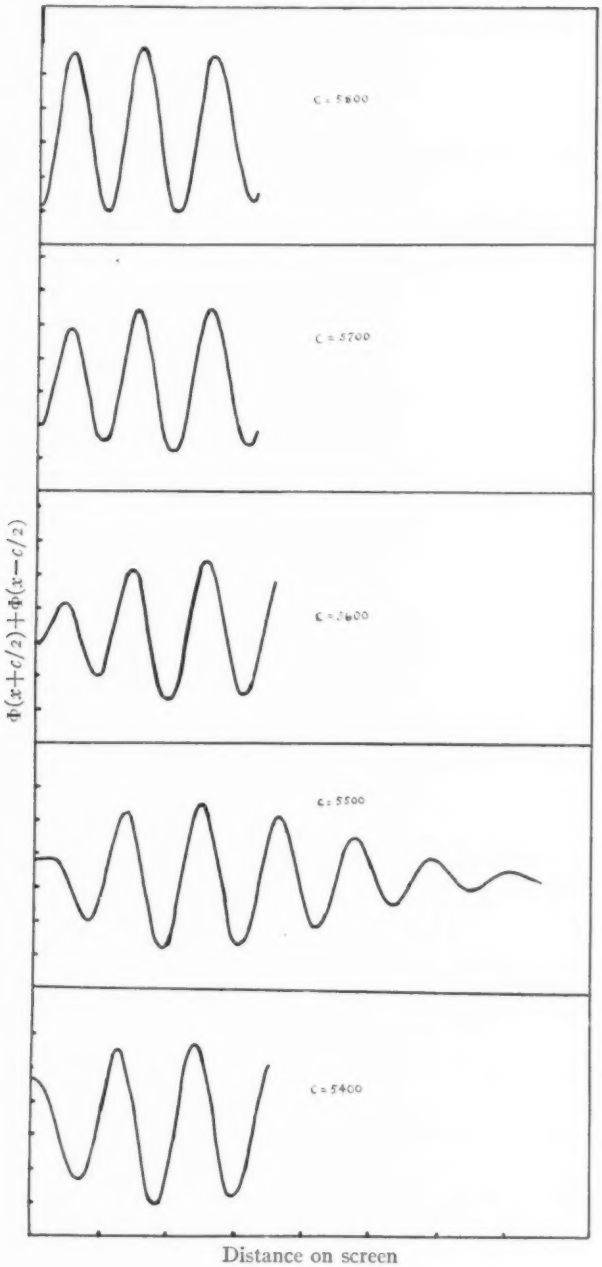


FIG. 3

parts of the field of view in the several patterns, and that c will accordingly have values differing with the different parts of the field observed. For instance, if the oscillations studied are those at the center of the field, we find a value of c corresponding to a wave-length 5581.0 A.

Because of the difficulty of finding a criterion for minimum visibility in the composite curves just discussed, resort was had to a mechanical means of superposing the two patterns in a manner that simulates the phenomena actually observed in an interferometer. The method was much the same as that previously employed by Sparrow.[1] The luminosity curves of Figure 1 were plotted on black paper and mounted on glass slides. Two identical slides were made of each curve, and were mounted one immediately above the other, so that they could be relatively displaced. White light from a uniformly illuminated background sends through the uncovered portions of the slides an amount of luminous energy proportional to that given by equations (17) and (18), and is focused by a cylindrical lens into a system of fringes on the ground glass of a camera. Observations were made visually as the slides were gradually displaced. The patterns could be easily photographed by introducing a plateholder. (See Plates VII and VIII.) Since the eye is more sensitive than the photographic plate to such phenomena, all quantitative results are based on visual settings.

The slides were about 12 by 50 cm. The displacements were measured by means of a vernier, reading directly to 0.01 inch. The cylindrical lens was of about 6-inch focus, and replaced the regular lens of the camera. The distance between the lens and the illuminated slides was about 6 meters.

Black paper was finally abandoned in making the patterns in favor of copper foil, in order to eliminate effects due to humidity. The foil was secured to the glass with shellac, after which the curves were plotted with a special device, and the traces were carefully cut. All observations in Table II were made from copper patterns, while those in Table III were from the paper patterns, since the accuracy attainable with finite slits did not seem to warrant the time required to repeat the observations with copper patterns.

[1] *Astrophysical Journal*, **44**, 76, 1916.

Tables II and III contain the effective wave-lengths as determined by the visual method. In Table II each value recorded is the mean of five settings. A series of five such sets of observations for each temperature was taken by Professor Hoxton, Messrs. J. B. Ball, M. Dick, W. T. Straley, and the writer. The mean for each temperature for all observers thus represents 125 determinations of the wave-length. A series of five settings occupied usually about fifteen minutes. On account of fatigue it was found advisable to take not more than two or three series of readings the same day. For the very narrow slits (Table II) the accuracy comes well within the bounds set by Anderson.

TABLE III

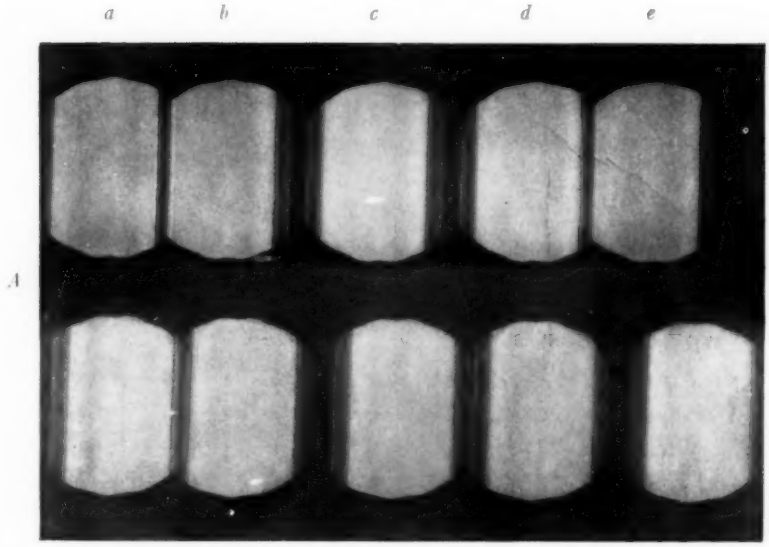
EFFECTIVE WAVE-LENGTH (VISUAL METHOD)

Observer	FINITE SLITS	
	Temperature 6800°	
	Ratio $a : b = \frac{1}{4}$	Ratio $a : b = \frac{1}{2}$
H. L. A.....	5492.8 ±12.	5207.6 ±20.
F. A.....	5512.5 ± 4.1	5327.7 ±13.
C. W. B.....	5446.8 ± 1.9	5067.9 ± 7.0
J. S. M.....	5420.7 ±15.	5156.0 ±40.
Mean.....	5478.2 ±14.6	5189.8 ±36.6

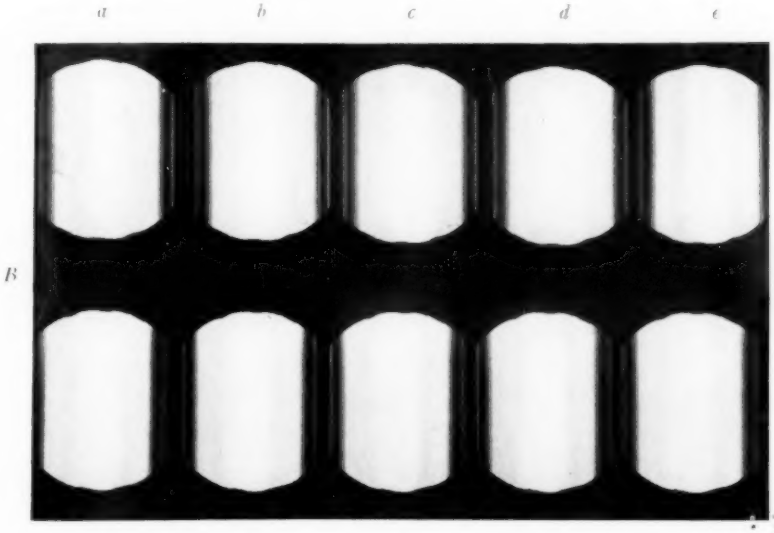
In Figure 4 the average values of the effective wave-lengths in Table II are plotted against the reciprocal of the temperatures. Circles of radii equal to the probable errors are described about each point. The relation is very nearly linear.

For finite slit-widths the readings, as one would predict from theory, became increasingly discordant for ratios a/b equal to $\frac{1}{4}$ and $\frac{1}{2}$ (Table III). The fringes for ratios employed may scarcely be said to approach disappearance, certainly not for the ratio $\frac{1}{2}$. The degree of the extinction of the fringes for different slit-widths is shown in the accompanying photographs (Plates VII and VIII).

The results show that for wide slit-widths the effective wave-length is somewhat indeterminate, or rather that no such definite quantity exists. For very narrow slits, however, it is found that



Temperature, 6800° K. Infinitely narrow slits.



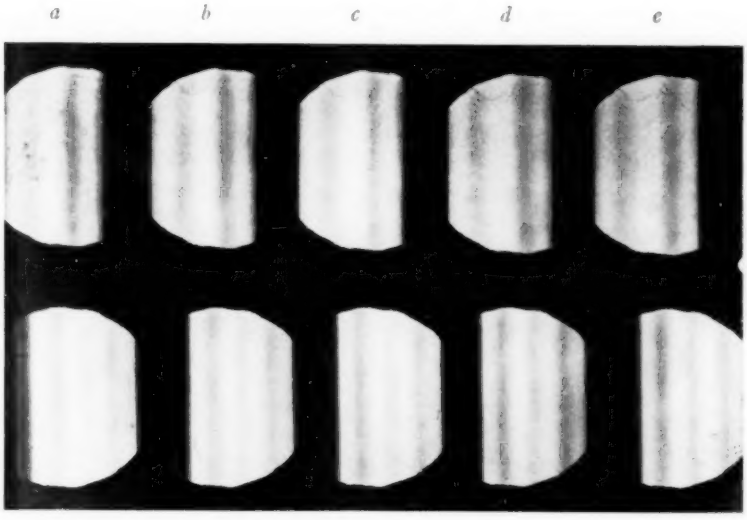
Temperature, 6800° K. Finite slits, $a/b = 1/4$

PHOTOGRAPHS OF FRINGES FORMED BY CYLINDRICAL LENS SYSTEM

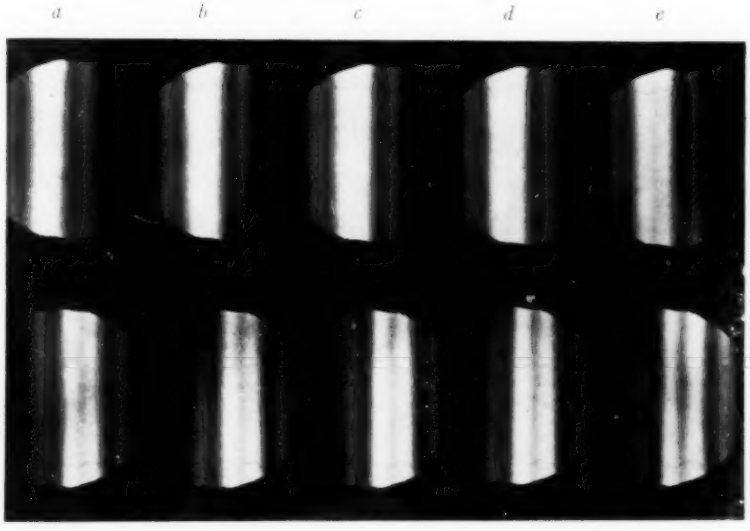
A.—Ten exposures showing appearance of fringe system in passing through first minimum of visibility in intervals of 0.01 inch (10 A). The successive displacements are indicated by the letters, the minimum being in the *e*–*f* region.

B.—Made under the same conditions as *A*. The failure of the fringes to vanish in the case of the finite slits is more marked than for the very narrow slits.

PLATE VIII



Temperature, 6800° K. Finite slits, $a/b = 1/4$



Temperature, 6800° K. Finite slits, $a/b = 1/2$

PHOTOGRAPHS OF FRINGES FORMED BY CYLINDRICAL LENS SYSTEM

A.—Photographs at slightly greater intervals, 0.05 inch in passing through the first minimum region, which is near E.

B.—Region of first minimum of visibility, at successive displacements of 0.05 inch. Note the failure of the fringes even to approach disappearance.

there is a definite wave-length associated with the particular phenomenon observed. Since in the recent astronomical investigations the apertures are small, relative to the distance separating them, the results in this paper have practical application. Time did not permit carrying out the computations for ratios smaller than $\frac{1}{4}$. From a consideration of the results before us, however, there seems to be no doubt that for smaller ratios the settings on minimum visibility would be with greater certainty, and the effective wave-length would have a correspondingly more definite value.

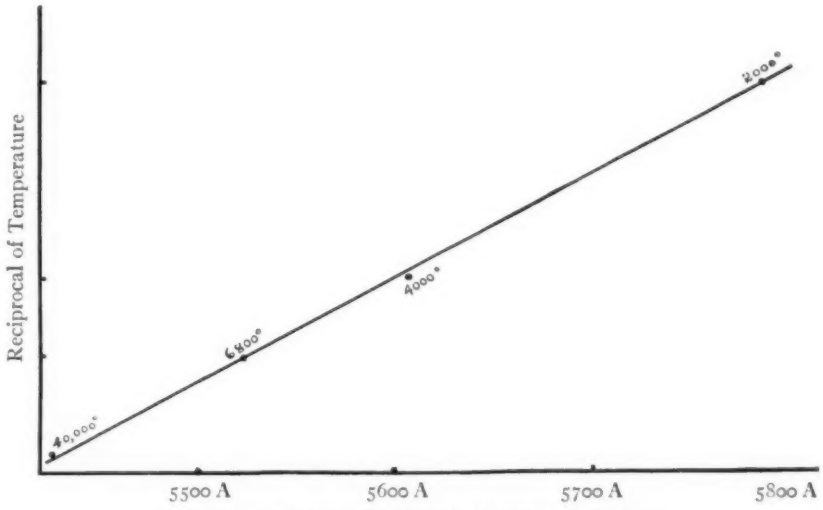


FIG. 4.—Effective wave-lengths (visually determined)

It should be noted that the interference pattern from a single source is obtained from theory, and that the compounding of the two sources is put on an experimental basis.

From the results of our investigations we may now, following Anderson,[1] outline the method of finding the effective wave-length of any double star whose effective temperature corresponding to its spectral distribution is known.

Integral (17) is evaluated for a number of temperatures. These computations are somewhat laborious but they are made once for all. Patterns of copper foil are made from the computed results

[1] *Astrophysical Journal*, **55**, 48, 1922.

corresponding to each temperature. These are mounted and relatively displaced until the first minimum of visibility of the fringe system is obtained, one setting to the right and one to the left of the line of centers. The difference between the settings gives the value of the effective wave-length. The average of many such determinations is the value adopted. A graph is then constructed in which these values of the effective wave-length are plotted against the reciprocal of the absolute temperature (Fig. 4), from which the effective wave-length of a star of any effective temperature may at once be found.

To illustrate, suppose that it is desired to find the effective wave-length of an Fo-type star. We look up the effective temperature of the Fo-type star and also the effective temperature of a star of the type of the sun, both temperatures being known from the spectral distribution of the respective stars. We immediately read from our graphs the effective wave-lengths of the Fo-type star and the star of the type of the sun. Denote these, respectively, by λ_1 and λ_2. Anderson[1] has made laboratory measures of the effective wave-length of sunlight by illuminating an artificial double star with sunlight reflected from a freshly polished mirror. Denote this value by λ_0. The effective wave-length λ of the Fo-type star will then be given by

$$\lambda = \lambda_0 + [\lambda_1 - \lambda_2]. \qquad (22)$$

The quantity in brackets is determined by our method. This quantity being relative, the absolute values of λ_1 and λ_2 are not required. For this reason, any error due to the assumption of black-body radiation does not sensibly affect the result. This would mean graphically that the curves representing absolute and relative effective wave-lengths would be approximately parallel, so that differences in the two cases would be for all purposes the same.

It is assumed that the absolute effective wave-length of sunlight is known under the same atmospheric and zenith conditions as those under which the observations are made on the star. Anderson indicates the solution of this problem.

[1] *Astrophysical Journal*, **55**, 48, 1922.

The method applies also to a double star whose components are of unequal intensities and temperatures. Some preliminary observations were taken with patterns representing such components. While it is more difficult to recognize the minimum of visibility, yet the same difficulty will be met when fringes from such a double star are viewed through an interferometer, since the fringes produced by the copper-foil patterns will simulate what appears in the interferometer.

It is worth noting (compare Tables I and II) that within the range of accuracy usually attainable the expression in brackets in equation (22) may be replaced by the distance to the first minimum (λ_x). In the visual determinations the brackets for temperatures 2000°, 4000°, 40,000° take the values 264.9 A, 84.9 A, 96.8 A. For the same temperatures when λ_x is used, the values become 266.8 A, 81.9 A, 92.3 A, changing the value in equation (22) only by the respective amounts +1.9 A, −3.1 A, −4.5 A. The use of λ_x would very materially reduce the labor in obtaining points for the wave-length-temperature curve, since the work of preparing patterns and taking visual observations would be eliminated. Further study is necessary before the statement could be made that such a simplification is possible with heterogeneous components of a double star.

This work was suggested by Professor C. M. Sparrow, to whom I am indebted also for advice and encouragement during the progress of the work. I wish also to thank Professor Hoxton for his interest and encouragement in many ways.

Acknowledgments are due to Professors H. L. Alden and J. S. Miller, Jr., for taking observations; also to Mr. C. W. Bruce, who contributed much time to observing and computing. I desire to thank Mr. A. J. Weed, mechanician, for help and co-operation.

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August 9, 1923

THE ASYMMETRY IN STELLAR MOTIONS AND THE EXISTENCE OF A VELOCITY-RESTRICTION IN SPACE[1]

By GUSTAF STRÖMBERG

ABSTRACT

Observational evidence of a velocity-restriction in space.—The asymmetry in the distribution of stellar velocities, which has been found by several investigators, has been studied, and it is found that a very definite relation exists between group-motion and the internal motion for different classes of objects. This relation, which is expressed by a quadratic equation, holds for all classes of objects for which we have data regarding their velocities. This dependence of group-motion upon internal motion in the velocity-groups leads to the representation of stellar velocities as a product of two three-dimensional frequency-functions F_1 and F_2, symmetrical around two different centers. The sun's velocity relative to the center of F_1, which itself is a sum of several concentric ellipsoidal frequency-functions, is found to be $\alpha = 261^\circ.6$, $\delta = +14^\circ.6$, $v = 13.9$ km/sec. The center of the second function F_2 cannot be determined directly, but by assuming that the stars in group X, XI, and XII of Table I have ultimate velocities, the center of F_2 can be found. The sun's velocity relative to the center of the frequency-function F_2 is $\alpha = 323^\circ$, $\delta = +60^\circ$, $v = 300$ km/sec.

The function F_1 can be interpreted as representing the distribution law of the velocities in the local system of stars in the absence of an external effect. F_2 can be interpreted as an external velocity-restriction in a fundamental frame of reference, which seems to be the same for near and distant objects.

The consequences of the existence of two frequency-functions are first derived deductively; later an inductive study is made.

Physical aspects of the theory.—The present study indicates that the distribution of stellar motion is determined by two connections, one with the local system and another with a very large cosmical system. The physical nature of the second connection is uncertain. The action of the larger system is similar to that of a field of stars or dust. This field may also be an absolute property of space.

INTRODUCTION

The proper motions and radial velocities of the stars show a certain asymmetry in their distribution, which is generally explained as a reflection of the sun's motion relative to the stars in general. When the stellar velocities are referred to the centroid of a large number of stars, this asymmetry is nearly eliminated. We can thus determine the sun's velocity relative to the stars and we find its value to be about 20 km/sec. toward the apex $\alpha = 270^\circ$, $\delta = +30^\circ$. Kapteyn found in 1904 that the proper motions of the stars indicated the existence of two interpenetrating star streams. Schwarzschild[2] showed that the distribution law of stellar motions could be

[1] *Contributions from the Mount Wilson Observatory*, No. 275.

[2] *Göttingen, Nachrichten*, p. 191, 1908.

represented by an ellipsoidal frequency-function, and Charlier[1] found that the deviation from an ellipsoidal distribution-law could not be reduced by assuming two star streams in accordance with Kapteyn's suggestion. The deviations pointed to a skewness in the distribution together with an excess of stars of large linear velocities. It was noted by Adams and Kohlschütter[2] in 1913 that the large radial velocities for stars in the Northern Hemisphere, after correction for the sun's velocity, were overwhelmingly negative. B. Boss[3] computed the space-velocities of a number of stars of measured parallaxes and found that the high-velocity stars showed a marked tendency to move toward the region in the sky limited by galactic longitudes 140° to 340°. The same effect was independently found by Adams and Joy[4] from a study of the space-velocities of stars of high radial velocity. From an investigation of the space-velocities of 1300 stars of late spectral types the writer[5] found that about 100 stars of space-velocity larger than 100 km/sec. had their apices between galactic longitudes 143° and 334°, and *not a single one* between longitudes 334° and 143°. It was furthermore found that this asymmetry in the velocity-distribution, which remained after the velocities had been referred to the centroid of the stars, could be traced even among stars of ordinary speed, producing a difference between the most frequent velocity-vector and the mean velocity. The same skewness in the velocity-distribution was found by Boss, Raymond, and Wilson[6] from a study of the space-velocities of 520 stars of known parallaxes.

The line of asymmetry in the distribution of velocities is nearly opposite to the sun's motion relative to stars of ordinary speed. The sun's velocity is thus dependent upon the speed of the stars to which its motion is referred, although the apex of the solar motion is only slightly affected.[5] This latter circumstance makes it diffi-

[1] *Meddelanden från Lunds Observatorium*, Serie II, No. 9, 1913, pp. 92 ff.

[2] *Mt. Wilson Contr.*, No. 79; *Astrophysical Journal*, **39**, 341, 1914.

[3] *Popular Astronomy*, **26**, 686, 1918; *Publications of the American Astronomical Society*, 22d Meeting, 1918.

[4] *Mt. Wilson Contr.*, No. 163; *Astrophysical Journal*, **49**, 179, 1919.

[5] *Mt. Wilson Contr.*, No. 245; *Astrophysical Journal*, **56**, 265, 1922.

[6] *Astronomical Journal*, **35**, 26 (No. 820), 1923.

cult to detect the phenomenon from a study of proper motions, which give only the direction of the sun's motion when the distances of the stars are not known. It makes the use of parallactic motion for computing mean parallaxes somewhat misleading, if the same solar velocity is used for stars of small and high velocity. The asymmetry seems to be the same for giant stars and dwarf stars and independent of spectral type, although it is most conspicuous among stars of high velocity. It has recently been found to exist in a very marked degree among stars of spectral types Me and variable stars of short period.

TENTATIVE EXPLANATION OF THE ASYMMETRY

The asymmetry in the distribution of stellar velocities seems thus to be a perfectly general phenomenon, dependent upon the speed alone, and to be independent of the physical nature of the objects concerned. It was first thought that it might be due to the excentric position of the sun in our local system of stars, whose center as defined by the B stars[1] is in galactic longitude 236°. Thus high speeds in a certain direction might be attained by stars falling toward the center of the local system. It was soon realized, however, that, if the stellar system has reached an approximately steady state, stars coming from the opposite side of the center should be equally numerous and have the same high speed, but in the opposite direction when passing our region in the local cluster. Furthermore, the effect in this case would vanish for stars moving perpendicularly to the direction toward the center.

Oort[2] suggested that there may be two or more distinct groups of stars having different group-motions and that the solar motion thus would come out differently as reference is to one or other of these groups.

It occurred to the writer that the asymmetry might be explained if for some reason high velocities referred to a certain reference-system in space were improbable. Thus if a star were moving toward the first quadrant of galactic longitudes (cf. the diagram in

[1] Charlier, "The Distance and the Distribution of the Stars of Spectral Type B," *Meddelanden från Lunds Observatorium*, Serie II, No. 14, 1916.

[2] *Bulletin of the Astronomical Institutes of the Netherlands*, No. 23, 1922.

Mt. Wilson Contr., No. 245, p. 20),[1] its velocity, referred to another origin in the third quadrant, might possibly be too great to be probable, whereas, if directed toward the third quadrant, its velocity referred to the origin mentioned would be smaller and in the realm of values occurring more frequently. Attention was then directed to the velocities of distant objects, and it was found that the translation of our local system, referred to the frame for which the "velocity-restriction" seems to exist, is the same as that derived from non-galactic nebulae and globular clusters, at least in so far as it was possible to determine this translation from observations now available.

To put these conceptions into mathematical form, we will assume that all the objects in our local system of stars form a unity, all parts of which have a common group-motion when no external "velocity-restriction" is acting. As an origin we shall use the sun, since all our measurements of proper motions and radial velocities are referred to it.[2] Since the collection of data is very heterogeneous with regard to speed, we assume the following general form of the frequency-function for the velocity-components referred to the element $dx\,dy\,dz$ of velocity-space:

$$F_1 = \frac{1}{\pi^{\frac{3}{2}}} \sum_{\nu} N_\nu l_\nu m_\nu n_\nu e^{-l_\nu^2(x-\alpha)^2 - m_\nu^2(y-\beta)^2 - n_\nu^2(z-\gamma)^2}. \qquad (1)$$

This function represents a sum of concentric and coaxial ellipsoidal velocity-distributions. N_ν is the number of objects belonging to each one of the component frequency-functions; l_ν, m_ν, n_ν are inversely proportional to the dispersions (square root of the mean of the squares of the velocities) along the principal axes; and α, β, γ represent the velocity-components, relative to the sun, of the common center of the component frequency-functions. The co-ordinate system used has its origin in the sun and its axes along the common principal axes of the velocity-ellipsoids.

[1] *Astrophysical Journal*, **56**, 284, 1922.

[2] A short account of this investigation has been given in *Mt. Wilson Comm.*, No. 84; *Nat. Acad. Proceedings*, **9**, 312, 1923. The origin used there is one relative to which the sun has a velocity of 20 km/sec. toward the apex $\alpha = 270°$, $\delta = +30°$.

The additional velocity-restriction is represented by the frequency-function

$$F_2 = e^{-k^2[(x+x_0)^2+(y+y_0)^2+(z+z_0)^2]}, \tag{2}$$

which is supposed to "act" simultaneously with F_1 upon the stars within the same element of velocity-space.

This function corresponds to a spherical distribution-law. The velocity (x_0, y_0, z_0) is the velocity of the sun in the fundamental system of reference. The actual probability F of a certain velocity (x, y, z) can then be found by multiplication of the two probabilities F_1 and F_2.[1] The function F can be written in the following form:

$$\left.\begin{aligned} F &= \sum_\nu A_\nu e^{-h_1^2(x-c_1)^2-h_2^2(y-c_2)^2-h_3^2(z-c_3)^2} \\ h_1^2 &= l_\nu^2+k^2 \qquad c_1 = -2k^2a_\nu^2(x_0+\alpha)+\alpha \qquad a_\nu = \frac{1}{\sqrt{2}\,h_1} \\ h_2^2 &= m_\nu^2+k^2 \qquad c_2 = -2k^2b_\nu^2(y_0+\beta)+\beta \qquad b_\nu = \frac{1}{\sqrt{2}\,h_2} \\ h_3^2 &= n_\nu^2+k^2 \qquad c_3 = -2k^2c_\nu^2(z_0+\gamma)+\gamma \qquad c_\nu = \frac{1}{\sqrt{2}\,h_3} \end{aligned}\right\} \tag{3}$$

The function F represents a sum of coaxial ellipsoidal distribution-laws, whose centers are shifted in a direction nearly opposite to that of the translation of the adopted origin in our fundamental reference-system. The amount of the shift increases with the internal motion in the different groups. The center of the distribution represented by this function has two limits. For stars of small relative motions, for which l, m, and n are large compared with k, the limit is

$$c_1 = \alpha, \qquad c_2 = \beta, \qquad c_3 = \gamma\,.$$

For stars of extremely large relative velocities the co-ordinates of the center approach the limit

$$c_1 = -x_0, \qquad c_2 = -y_0, \qquad c_3 = -z_0,$$

[1] A simple illustration of the use of a product of two frequency-functions, both referred to the same velocity-element, $dxdydz$, is offered by individuals walking on shipboard in the presence of a strong head-wind. The two limits correspond to different classes of individuals with infinitely strong and infinitely weak connection with the ship, the latter group being statistically at rest in the air.

which represents the center of the fundamental reference-system if referred to the sun as origin.

This shift in the center of the velocity-distribution with increasing internal velocities shows clearly for *all classes of celestial objects* for which we have data regarding their velocities.

COMPARISON BETWEEN THEORY AND OBSERVATION

In Table I are collected data that are representative of our present knowledge of stellar velocities. The constants of ellipsoidal frequency-functions for twelve different groups of objects are given in the eighth to the sixteenth columns. The quantities c_1, c_2, and c_3 are the co-ordinates of the centers of the velocity-groups referred to the sun as origin. The system of co-ordinates used is the galactic system with the x-axis toward the intersection of the galactic plane with the equator (in Aquila), the y-axis toward 90° galactic longitude, and the z-axis toward the north galactic pole, for which the adopted co-ordinates are

$$\alpha = 190^{\circ}.6, \qquad \delta = +27^{\circ}.2, \qquad 1900.0 \quad \text{(Kapteyn)}.$$

The equations for finding equatorial from galactic co-ordinates are then

$$\begin{aligned} \xi &= +0.1846\,x + 0.4494\,y - 0.8740\,z \\ \eta &= -0.9828\,x + 0.0844\,y - 0.1642\,z \\ \zeta &= \qquad\quad + 0.8893\,y + 0.4573\,z \end{aligned}$$

The dispersions along the principal axes are denoted by a, b, and c. The quantity a is the dispersion along that principal axis which most nearly coincides with the x-axis, and b and c are the dispersions along the principal axes that are nearest to the y- and z-axes. The values given for the dispersions are in all cases corrected for the systematic effect of accidental errors in the radial velocities, proper motions, and parallaxes.

The directions of the principal axes of the velocity-ellipsoids are given in the last three columns; L_1 is the galactic longitude of the axis along which the dispersion is equal to a, and B_1 the galactic latitude of the same axis. B_2 is the galactic latitude of the axis along which the dispersion is equal to b. The probable errors of

TABLE I

Group	Objects	No.	$V_\odot$	$\alpha_\odot$	$\delta_\odot$	K	c_1	c_2	c_3	a	b	c	L_2	B_2	B_3
			km			km	km	km	km	km	km	km			
I.....	Ursa Major group	22	18.5	127°.8	+ 40.0	0	+ 12.6	− 7.6	− 11.2	1	1	1	.....	.....	.....
							± 1	± 1	± 1	± 1	± 1	± 1			
II.....	Taurus group	39	45.6	271.8	− 6.9	0	− 44.7	+ 8.0	− 3.7	1	1	1	.....	.....	.....
							± 1	± 1	± 1	± 1	± 1	± 1			
III....	B stars	284	22.1	286.2	+ 25.5	+ 4.3	− 19.8	− 9.3	− 2.6	5.6	5.6	3.5	.....	(0°)	(0°)
							± 1	± 1	± 1	± 1.0	± 1.0	± 0.5			
IV.....	A stars (Central group)	230	14.2	271.7	+31.0	0	− 12.0	− 5.6	− 5.0	20.2	6.1	5.4	158°	− 6°.8	−22
							± 6.1	± 0.8	± 0.5	± 3	± 1	± 1	± 22	± 4.2	± 9
V.....	Stars of moderate velocity of spectral class F, G, K, M	768	15.2	262.1	+21.9	0	− 13.4	− 3.0	− 6.6	22.7	16.6	10.5	165.4	− 0.3	+ 2.7
							± 0.8	± 0.8	± 0.5	± 0.9	± 0.8	± 0.6	± 4.0	± 1.4	± 2.4
VI.....		510	32.4	288.8	+46.0	0	− 22.3	− 22.2	− 7.8	42.1	22.1	27.8	155.3	+ 1.1	−12.7
							± 3.9	± 2.6	± 2.4	± 2.5	± 1.1	± 1.9	± 4.6	± 7.6	±12.5
VII....	Bright-line nebulae	101	30.2	288.9	+43.5	+ 1.8	− 21.7	− 19.9	− 6.7	37.4	34.0	34.0	(163)	(−3)	.....
						± 2.7	± 4.4	± 6.1	± 7.7	± 2	± 2	± 2			
VIII...	M1e to M6e	47	41.3	288.7	+30.3	0	− 35.3	− 20.8	− 5.1	49.8	30.9	40.2	164	− 2	+ 5
							± 5.4	± 4.0	± 4.6	± 6	± 4	± 5	± 5	± 3	± 3
IX.....	Short-period variables	26	109	306	+47	0	− 67	− 85	− 8	69	69	69	.....	.....	.....
							± 20	± 19	± 16	± 10	± 10	± 10			
X.....	Stars of high velocity	21	268	324	+58	0	−102	−247	− 17	206	82	78	167	+ 5	+ 3
			± 27				± 46	± 22	± 19	± 80	± 20	± 20	± 5	± 3	± 3
XI....	Globular clusters	16	266	320	+64	0	− 90	−247	− 43	113	113	113	.....	.....	.....
			± 53				± 33	± 55	± 47	± 30	± 30	± 30			
XII....	Non-galactic nebulae	43	401	304	+81	+689	−138	−358	−116	359	359	359	.....	.....	.....
			±102			± 71	±144	± 97	± 78	± 50	± 50	± 50			

these quantities are given below the numbers to which they refer, and in several cases are only estimates.

Referred to the principal axes and to the center whose galactic co-ordinates are c_1, c_2, and c_3, the frequency-function of the velocities for each group can be written

$$F=\frac{N}{(2\pi)^{\frac{3}{2}}abc}\,e^{-\frac{x_i^2}{2a^2}-\frac{y_i^2}{2b^2}-\frac{z_i^2}{2c^2}}.$$

The sun's velocity V_0 and the position of its apex as referred to the twelve different groups are given in the fourth to the sixth columns of Table I. These data are connected with the quantities c_1, c_2, and c_3 by the relations

$$\begin{aligned} V_0 \cos \alpha_0 \cos \delta_0 &= -\xi \\ V_0 \sin \alpha_0 \cos \delta_0 &= -\eta \\ V_0 \sin \delta_0 &= -\zeta \end{aligned}$$

where ξ, η, and ζ are the values of c_1, c_2, and c_3 expressed in equatorial co-ordinates. The systematic correction applied to the measured radial velocities is equal to $-K$, where K is the quantity given in the seventh column.

The elements for the Ursa Major group are those given by Bottlinger and Rasmuson.[1] For the Taurus group the elements computed by Boss[2] are given. The dispersion in these two groups has been assumed to be about 1 km/sec. For the B stars (Bo to B5) Gyllenberg's[3] elements for the solar motion have been used. The elements for the A stars are those for the Central group in *Contribution* No. 257,[4] referred to the sun and corrected for the systematic effect of accidental errors in the quantities involved.

For stars of spectral types F, G, K, M, a special computation was made, based upon the three velocity-components for stars of known proper motion, parallax, and radial velocity. In order to conform with the adopted frequency-function the distribution was represented by a sum of two ellipsoidal frequency-functions,

[1] *Meddelanden från Lunds Observatorium*, Serie II, No. 26, 1921.

[2] *Astronomical Journal*, **26**, 31 (No. 604), 1908.

[3] *Meddelanden från Lunds Observatorium*, Serie II, No. 13, 1915.

[4] *Astrophysical Journal*, **57**, 77, 1923.

and the method devised in *Contributions* Nos. 251[1] and 257 was employed. One modification of the method was introduced, however. The equations of condition for determining the twenty constants involved are of the form

$$\frac{dn}{dN_1}\Delta N_1+\frac{dn}{dh_1}\Delta h_1+ \ldots . +\frac{dn}{dN_2}\Delta N_2+\frac{dn}{dh_1'}\Delta h_1'+ \ldots . = n_o-n_c ,$$

where N_1 and N_2 are the total number of stars in the two frequency-functions, the sum of which represents the actual distribution. These equations of condition were given additional weights inversely proportional to the square roots of the computed number (n_c) of velocity vectors terminating in the different parallelopipeds of the velocity-space.

The solar motion as based upon the bright-line nebulae was computed from the radial velocities measured by Campbell and Moore.[2] The direction of the axis of preferential motion was assumed to have the co-ordinates $\alpha = 90°$, $\delta = +13°$. Six nebulae of excessively high velocities were omitted.

In group VIII are given the elements of the velocity-distribution for forty-seven long-period variable stars of spectral type M1e to M8e. The data are from an investigation by Mr. Merrill and the writer published as *Contribution* No. 268.[3]

Group IX is based upon the radial velocities of twenty-six variable stars of period less than one day. These velocities have all been determined by Messrs. Adams and Joy, and I am under great obligation to them for their courtesy in allowing me to use their data in advance of publication.

Group X consists of twenty-one stars of very high velocity, which in my previous study of velocities seemed to form a separate group. They are mostly of spectral class F and have space-velocities ranging from about 200 to 600 km.

For the globular clusters and non-galactic nebulae the radial velocities alone can be used. In most cases these have been determined at the Lowell Observatory by Professor Slipher, through

[1] *Astrophysical Journal*, **56**, 265, 1922.

[2] *Lick Observatory Publications*, **13**, Part IV, 1918.

[3] *Astrophysical Journal*, **59**, 148, 1924.

whose courtesy several unpublished velocities could be included. The solar motion for the globular clusters has been computed by Lundmark.[1] The designation "non-galactic nebulae," as used by Hubble, has been adopted here; twenty-four of the forty-three objects are known to have spiral structure.

As seen in Table I, the grouping is based upon motion for groups I, II, and X, and upon other physical criteria for the other groups. For groups V, VI a statistical dissection into two velocity-groups has been performed. In several of the groups it was found that an asymmetrical distribution of velocities was noticeable, even after the velocities had been referred to the centroid chosen for the group. This asymmetry in the individual groups always presented itself as a tendency for the fastest moving stars in the group to move toward galactic longitudes 170° to 330°. A statistical dissection of the whole collection of velocities is out of the question, since we do not know the space-velocities for all the objects involved. Furthermore, the grouping on the basis of physical properties (apart from motion) is very convenient, and may give us valuable information concerning the motion of different classes of objects.

TABLE II

Groups	No.	V_0	α_0	δ_0	c_1	c_2	c_3
		km			km	km	km
IV, V.....	998	15.1	267°.1	+25°.3	−13.3 ± 0.8	− 4.3 ± 0.8	−5.8 ±0.5
VI, VII...	611	32.0	288.8	+46.0	−22.0 ± 3.5	−21.9 ± 2.5	−7.7 ±2.4
	a	b	c	L_1	B_1	B_2	
	km	km	km				
IV, V....	22.1 ± 0.9	13.1 ± 2.0	9.2 ± 0.6	164° ± 4	−1° ±1.5	+1° ±2	.....
VI, VII...	39.8 ± 2	24.5 ± 2	30.9 ± 2	155 ± 5	+1 ±7	−13 ±12	.....

The projection on the galactic plane of the different velocity-ellipsoids is shown in Figure 1 (p. 238). As several of the ellipsoids are nearly identical, groups IV and V were combined; and, similarly, groups VI and VII. The data for these combined groups are given in Table II.

[1] *Publications of the Astronomical Society of the Pacific*, **35**, 318, 1923.

The shift of the center of the velocity-distribution with increasing internal motion in the different groups is very clearly seen in

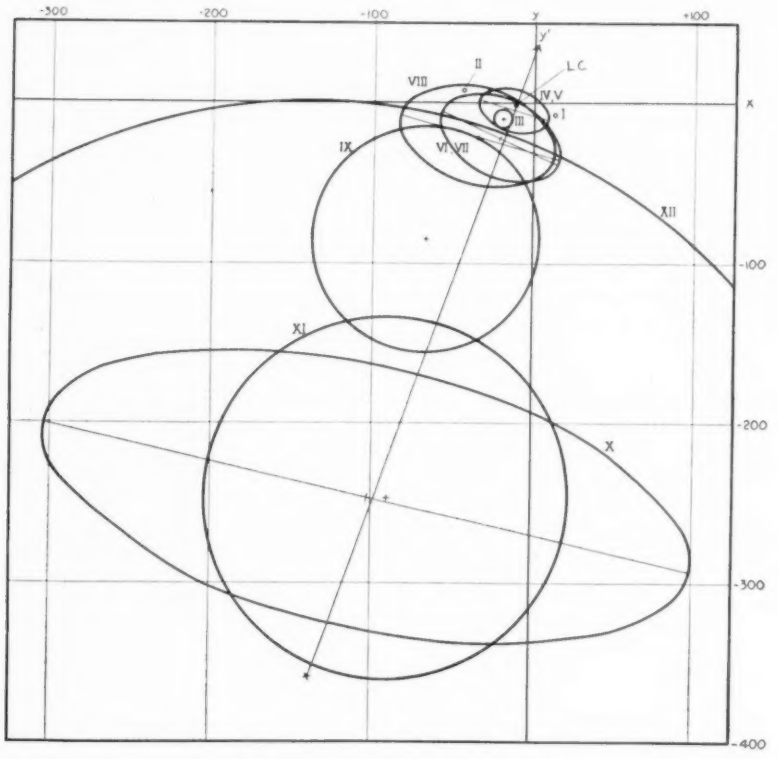

FIG 1.—Projection of the velocity-ellipsoids for different classes of stars on the galactic plane. The sun is at the origin and the vector from the origin to the center of the ellipses or circles represents the group-motion relative to the sun of the different groups of objects. The axes of the ellipses are equal to the dispersion in velocity within the groups. The systematic change in group-motion with increasing internal dispersion can be clearly seen. The point marked *L.C.* (limiting center) corresponds to the computed position of the center of the velocity-figure, where the dispersion vanishes. The line marked *y'* represents approximately the direction of translation of objects with small divergence in motion relative to objects of large divergence.

Figure 1. The direction of this shift in passing from slow- to fast-moving objects was found to be in galactic longitude $250°.7 \pm 5°$. The velocity-ellipsoids were then projected on a plane through the z-axis and through a line in galactic longitude $70°.7 - 250°.7$. This

new projection is shown in Figure 2. From this diagram the galactic latitude of the line of displacement of the center was estimated to $-5° \pm 3°$.

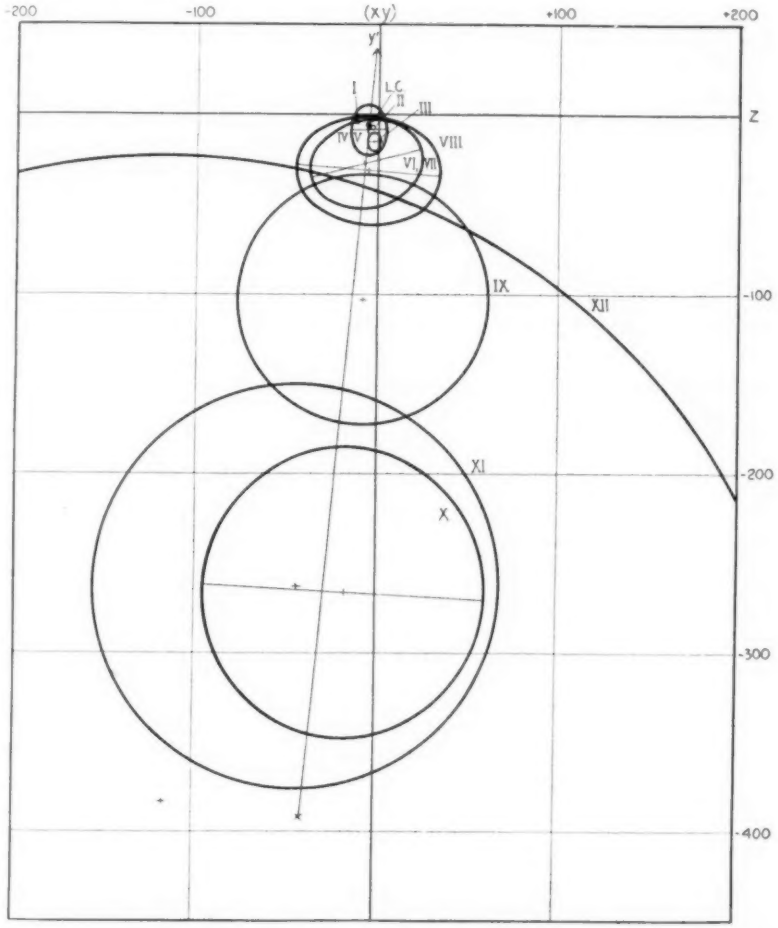

FIG. 2.—Intersection of the velocity-ellipsoids with a plane through the z-axis (toward the pole of the galaxy) and through a line in galactic longitude 70°.7. This line is marked (xy).

By introducing a new system of co-ordinates we can confine the systematic displacement of the center of velocity-distributions to one axis, the displacements on the other two axes being much

smaller and not systematic. The directions of the new axes (x' y' z') are

	L	B	α	δ
x'	340°.7	0°.0	271°.5	−17°.1
y'	70.7	+ 5.0	327.6	+61.2
z'	250.7	+85.0	188.9	+22.5

The x'-axis is approximately parallel to the direction of stream motion (major axes of the ellipsoids), the y'-axis is parallel to the systematic displacement of the centroids of the different velocity groups, when passing from groups of high to those of low dispersion, and the z'-axis points very nearly toward the north pole of the galaxy. The co-ordinates of the centers of the different velocity-ellipsoids referred to the sun and to the x'-, y'-, z'-axes are given in Table III.

TABLE III

Group	c'_1	c'_2	c'_3
	km/sec.	km/sec.	km/sec.
I	+14.4± 1	− 4.0± 1	−10.9± 1
II	−44.8± 1	− 7.5± 1	− 3.1± 1
III	−15.6± 1	− 15.5± 1	− 1.3± 1
IV, V	−11.1± 0.8	− 8.9± 0.8	− 5.0± 0.5
VI, VII	−13.5± 3.4	− 28.5± 2.6	− 5.2± 2.4
VIII	−26.4± 5.3	− 31.6± 4.2	− 2.4± 4.6
IX	−35 ± 20	−103 ± 19	+ 1 ±16
X	−15 ± 44	−267 ± 26	+ 6 ±19
XI	− 3 ± 36	−266 ± 53	−20 ±47
XII	−12 ±140	−392 ±103	−82 ±78
Mean of IV to XII	−11.6± 0.8	.....	− 5.0± 0.5

Taking into account probable errors, we can regard the values of c'_1 and c'_3 in Table III as fairly constant. The outstanding discrepancies are the values for the Ursa Major and the Taurus groups. These deviations are confined to the x'-co-ordinate, however, and are probably connected with the higher mobility along this axis.

The systematic change in the group-motion is now confined to the y'-axis, and is measured by the quantity c'_2. According to our hypothesis of the existence of two frequency-functions, we may

expect that the relation between c_2' and b will be that given in equations (3), namely,

$$\left.\begin{aligned} c_2' &= -pb^2+\beta' \\ \text{where} \quad p &= 2k^2(y_0'+\beta') \end{aligned}\right\} \tag{4}$$

The accents here indicate that the co-ordinates in equations (3) are referred to the (x', y', z') system of co-ordinates, the axes of which nearly coincide with the mean position of the principal axes of the velocity-ellipsoids.

The quadratic relation expressed by equation (4) holds with remarkable accuracy for all the groups except the non-galactic nebulae. Among the other groups only the B stars show a deviation larger than can be accounted for by the errors in the constants b and c_2'.[1] Omitting groups III and XII, we find the following relation:

$$\left.\begin{aligned} c_2' &= -0.0323\, b^2 - 5.7 \\ &\pm 0.0029 \quad \pm 0.7 \end{aligned}\right\} \tag{5}$$

Figure 3 (p. 242) shows this parabola, its axis coinciding with the y'-axes, and the agreement with the individual values of c_2' and b. The points representing the non-galactic nebulae are not indicated on the diagram. The observed value of b for these objects is 359 km/sec., whereas $c'_2 = -392$ (Table III) by equation (5) would give $b = 109$ km/sec.

The discrepancy for the non-galactic nebulae is, however, confined to the dispersion. The solar motion as computed from these nebulae is about the same as from the fast-moving stars in group X and the clusters in group XI, if account is taken of the probable errors in the respective quantities.

The systematic shift in the center of the velocity-ellipsoids with increasing speed can be found from other but less complete data than those used in Table I. The Cepheids in our local system are stars of high luminosity and small peculiar motion. Using the radial velocities of twenty Cepheids and an assumed position of

[1] There is reason to believe that the splitting up into component frequency-functions is artificial and that all the objects in groups IV to X form one unit with an asymmetrical velocity-distribution. The Taurus and the Ursa Major group, however, as well as the B stars, can be regarded as real "moving clusters," with systematic motion differing somewhat from that of the majority of the stars.

the apex of $\alpha = 270°$, $\delta = +30°$, we find a solar velocity of only 14.5 ± 2.0 km/sec. and an average residual radial velocity $\theta = 10.6$ km/sec. From twenty-four stars of spectral type F4 to G8 and of extremely high luminosity (omitting Cepheids and stars of similar spectra), the author finds from their radial velocities a solar velocity of 14.2 ± 1.9 and $\theta = 9.1$ km/sec. This is corroborative evidence that the slow-moving objects give a smaller

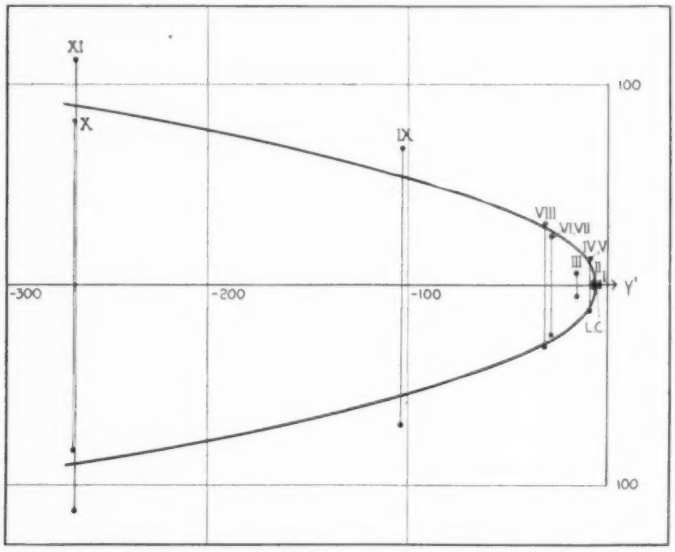

FIG. 3.—Relation between systematic change in group-motion relative to the sun (abscissae), and dispersion along an axis parallel to this systematic shift (ordinates). This relation is represented by a parabola with its axis along the line of systematic displacements.

velocity for the sun than stars of higher velocity. The B stars form an exception, as they give a solar motion of 22 km combined with a peculiar motion of only about 5 km. These stars seem to form a moving group of their own, for which the group-velocity along the y'-axis does not quite follow the simple law expressed by equation (5). In *Contribution* No. 267[1] it has been shown that the solar motion deduced from the radial velocities of the Me stars increases steadily with the residual radial velocities for the group of stars

[1] *Astrophysical Journal*, **59**, 97, 1924.

relative to which the sun's motion is referred. Using the approximate relations $c_2' = V_0$ and $b = 1/\pi/2\ \theta$, we find new points which all fall almost exactly on the parabola in Figure 3.

In Table III are given the weighted means of c_1' and c_3'. These values correspond to the quantities α and γ in equation (1), referred to the (x', y', z') system of co-ordinates. The corresponding value of β is that given in equation (5). These three quantities define one of the limits of the series of centers of the velocity-ellipsoids. According to equation (1), they give the common center of the velocity-ellipsoids in the absence of an external velocity-restriction.

The sun's velocity relative to this limiting point is

x' km	y' km	z' km	x km	y km	z km	α	δ	v km
+11.6	+5.7	+5.0	+12.7	+1.1	+5.5	261°.6	+14°.6	13.9
± 0.8	±0.7	±0.7	± 0.8	±0.7	±0.7			

The velocity of the sun is the same as that found from F and G stars of very great luminosity. This point, which we will call the "limiting center," coincides closely with the most frequent velocity found among stars of spectral type F, G, K, and M, as determined by the author,[1] and by Boss, Raymond, and Wilson.[2] The co-ordinates for this point as given by the author and referred to the sun are

$$x = -11.2, \qquad y = -4.7, \qquad z = -5.7 \text{ km/sec.}$$

The sun's velocity relative to this point, which can be found by reversing the signs, agrees well with the values given above.

The position of this limiting center is indicated by a dot in Figures 1, 2, and 3, and is marked *L.C.*

INDUCTIVE STUDY OF THE VELOCITY-DISTRIBUTION

The large amount of data collected in Table I makes it possible to refrain from using any hypothesis concerning the nature of the frequency-function, and to determine this function *a posteriori* from the observational data. We have studied different classes of celestial objects and determined their motions relative to the sun,

[1] *Mt. Wilson Contr.*, No. 245; *Astrophysical Journal*, **56**, 292, 1922.

[2] *Astronomical Journal*, **35**, 26, 1923.

using as reference frames "non-rotating" co-ordinate systems, i.e., systems in which the algebraic sums of the proper motions of a very large number of stars are zero, or, what amounts to the same, systems in which the proper motions of the stars in general are as small as possible.

The motions of the objects belonging to the Ursa Major group and the Taurus group, respectively, are very nearly the same, but the velocities differ more and more among themselves as we proceed to the later groups in Table I. At the same time the motion of the group as a whole increases systematically in a certain direction. For representation of the group-motion and of the divergence in motion of the individual members of the group we have used frequency-functions of the ellipsoidal type. This representation is known to be a close approximation for all the groups. The globular clusters are very unevenly distributed over the sky, however, and as we know only their radial velocities, we must make an assumption about the value of the K-term (systematic correction to the radial velocities) in order to find definite values of the constants. For the non-galactic nebulae the K-term can be directly determined, and, although of course the constants for these objects are rather uncertain, as is indicated by their probable errors, the residual velocities can be fairly well represented by an error-curve.

The constants of the velocity-ellipsoids have been determined for each class of objects. Referred to the principal axes, the frequency-function for each group can be written

$$\frac{N h_1 h_2 h_3}{\pi^{\frac{3}{2}}} e^{-h_1^2(x''-c_1'')^2-h_2^2(y''-c_2'')^2-h_3^2(z''-c_3'')^2}$$

$$h_1^2=\frac{1}{2a^2}, \qquad h_2^2=\frac{1}{2b^2}, \qquad h_3^2=\frac{1}{2c^2}$$

N is here the number of stars in the group; x'', y'', and z'' are the velocity-components of a star referred to the sun and to the principal axes of the particular velocity-ellipsoid. We have one such frequency-function for each group, and, as the principal axes for the different groups are nearly parallel, the velocities can be referred to a common system of co-ordinates without changing the form of the frequency-function.

Thus the frequency-function for the velocities of the whole collection of objects can be written

$$F=\sum_{\nu} A_{\nu} e^{-h_1^2(x'-c_1')^2-h_2^2(y'-c_2')^2-h_3^2(z'-c_3')^2}. \tag{6}$$

The values of h_1, h_2, and h_3 vary considerably for the different groups. Using the axes defined on page 240 we find that h_1 in general is smaller than h_2 and h_3, and h_2 nearly equal to h_3. This indicates a preferential motion along the x'-axis. The x'-co-ordinate of the center, c_1', is fairly constant, but deviates somewhat from the mean for the Ursa Major and the Taurus group; c_2' shows a very large variation (from 0 to -400 km); and c_3' is very nearly constant. The large variation in c_2' is found to be correlated with the dispersion of the velocities in the group, and the following empirical relation has been found to hold with a very high degree of accuracy,

where
$$\left.\begin{aligned} c_2' &= -pb^2+\beta' \\ p &= +0.0323 \pm 0.0029 \text{ sec./km} \\ \beta' &= -5.7 \pm 0.7 \text{ km/sec.} \end{aligned}\right\} \tag{7}$$

It is only for the B stars and the non-galactic nebulae that this relation does not hold within the uncertainties of the quantities involved.

Equation (7) can be written

$$c_2' = -\frac{p}{2h_2^2}+\beta'.$$

Inserting this value of c_2' in equation (6) we find, after a simple transformation,

$$F=e^{-p(y'-\beta')}\sum_{\nu} B_{\nu} e^{-h_1^2(x'-c_1')^2-h_2^2(y'-\beta')^2-h_3^2(z'-c_3')^2}.$$

As the variation in c_1' and c_3' is fairly small, we will replace them by their mean values α' and γ'.

Thus

$$F=e^{-p(y'-\beta')}\sum_{\nu} B_{\nu} e^{-h_1^2(x'-\alpha')^2-h_2^2(y'-\beta')^2-h_3^2(z'-\gamma')^2}. \tag{8}$$

The first part of the frequency-function F is thus found to be constant for all the groups, and another part is found to correspond to a symmetrical distribution around a mean velocity whose components are α', β', γ'. The frequency-function F is proportional to the probability of the occurrence of a velocity-vector, the components of which are x', y', and z'. That this probability can be expressed as a product of two functions of these simple types immediately suggests that there are two independent causes which contribute to the actual frequency-function. It is the existence of the factor $e^{-p(y'-\beta')}$, or rather the fact that p is not zero, that produces the systematic change in group-motion along the y'-axis. The frequency-function $e^{-p(y'-\beta')}$ alone represents an increase in probability of velocity-components along the negative y'-axis. This increase in probability is exponential, and should give an infinitely large probability for $y'=-\infty$. This of course would be counteracted by the fact that $e^{-k_2^2(y'-\beta')^2}$ becomes zero for $y'=-\infty$. The function outside the summation sign, however, can easily be transformed into an ordinary frequency-function.[1]

By subtracting and adding the expression

$$k_1^2(x'-\alpha')^2+k_2^2(y'-\beta')^2+k_3^2(z'-\gamma')^2$$

to the exponents in equation (8), we find

$$\left.\begin{aligned} F &= F_1F_2 \\ F_1 &= \sum_\nu C_\nu e^{-l_\nu^2(x'-\alpha')^2-m_\nu^2(y'-\beta')^2-n_\nu^2(z'-\gamma')^2} \\ F_2 &= e^{-k_1^2(x'+x_0')^2-k_2^2(y'+y_0')^2-k_3^2(z'+z_0')^2} \\ l_\nu^2 &= h_1^2-k_1^2, \qquad m_\nu^2=h_2^2-k_2^2, \qquad n_\nu^2=h_3^2-k_3^2 \\ x_0' &= -\alpha', \qquad y_0'=\frac{p}{2k_2^2}-\beta', \qquad z_0'=-\gamma' \end{aligned}\right\} \tag{9}$$

The two frequency-functions F_1 and F_2, whose product represents the distribution law for all the cosmic velocities studied with a fair degree of approximation, are both symmetrical functions, but the centers of symmetry are different. On account of the particular choice of axes used the line joining these centers is parallel to the

[1] Although this transformation facilitates the physical interpretation of the distribution-law, it is not essential for the description of this law.

y'-axis. The velocity-components x'_0, y'_0, z'_0 represent the sun's velocity relative to the center of the velocity-distribution F_2, and $-\alpha'$, $-\beta'$,$-\gamma'$ the sun's velocity relative to the center of the distribution represented by the function F_1.

In this way we have found that the assumption of the existence of two frequency-functions made in the beginning follows naturally from the observed facts that the systematic shift of the centers of the velocity-distributions along the principal axes is related to the dispersion by equations of the form given in equation (7). The inductive study shows us further that the function F_2 is not necessarily spherical but can equally well be ellipsoidal.

In the distribution-function F in equation (9) the velocities are referred to the axes given on page 240. For a group of stars of small relative velocities, i.e., one for which l, m, and n are large compared with k_1, k_2, and k_3, the function F_1 alone determines the velocity-distribution, and the limiting center of the distribution has the co-ordinates α', β', γ' relative to the sun. For stars of excessive velocities, for which l, m, and n are small compared with the k's, the function F_2 defines the velocity-distribution. The limiting center of the distribution F_2 has the co-ordinates $-x'_0$, $-y'_0$, $-z'_0$. In the particular system of co-ordinates used we have further

$$\left.\begin{aligned} \alpha' &= -11.6 \pm 0.8 \text{ km/sec.} \\ \beta' &= -\ 5.7 \pm 0.7 \\ \gamma' &= -\ 5.0 \pm 0.7 \end{aligned}\right\} \qquad (10)$$

The function F_2 produces a "velocity-restriction" in a co-ordinate system relative to which the sun has a velocity, whose components are x'_0, y'_0, z'_0.

The values of k_1, k_2, k_3 and of y'_0, which is a function of k_2, cannot be directly determined. It is only the *direction* of the vector on which y'_0 is situated that is given. It is further obvious that y'_0 must be larger than 267 km/sec. As the direction of the sun's velocity relative to fast-moving objects and relative to distant objects appears to be the same, we wish to define y'_0 as the sun's velocity relative to the most distant objects, whose velocities are known. The value of y'_0 for the globular clusters is $+266$ km

(probable errors less than 100), and for the non-galactic nebulae $+392\pm103$ km. As we have not yet found any class of objects with higher velocity and larger group-motion, we may regard $+300$ km as a probable value of y'_0. When this is combined with the previously given values of y'_0 and z'_0, we find that the sun's velocity is 300 km/sec. in the direction

$$L=68^\circ, B=+6^\circ;\ \alpha=323^\circ, \delta=+60^\circ,$$

with an uncertainty probably less than 10°.

It is rather unexpected to find a very pronounced stream-motion among the fast-moving stars in group X. On account of the small number of objects in this group, the existence of stream-motion can hardly be regarded as established in this case. The elongation of the velocity-ellipsoid is mainly due to two stars, Numbers 289 and 560 in *Cincinnati Publications*, No. **18**. The highest velocity known, on the other hand, is that of the two stars A. Oe. 14318 and 14320, which have a common motion of about 900 km/sec. toward the apex $L=259^\circ$, $B=-17^\circ$. These two stars were omitted in the present study, but their inclusion would increase the dispersion along the y'-axis. Regarding them as a single star, we thus find the following elements for group X:

$c_1=-100$	$c'_1=-\ 8$	$a=205$
$c_2=-278$	$c'_2=-299$	$b=130$
$c_3=-\ 30$	$c'_3=-\ 4$	$c=\ 96$

The dispersion is increased from 82 to 130 km, and the group-motion c'_2 from -266 to -299 km. The constants in the relation between displacement and dispersion have been recomputed, using the new elements for group X. Further, the Taurus group, the Ursa Major group, and the B stars have been omitted, as they form separate moving groups. The new relation is

$$c'_2=-0.0311b^2-6.2\ ,$$

which differs only slightly from equation (5). The velocity-ellipsoid for group X is now less elongated, however.

Using a value of $y'_0 = 300$ km/sec., we can determine the corresponding value of k_2 from equation (4). We thus find

$$k_2 = 0.00741 \text{ sec./km}$$

The corresponding values of k_1 and k_3 are arbitrary, but must be smaller than h_1 and h_3 for group X, which, if we use the revised values for the dispersion, are 0.00345 and 0.00737, respectively. It is possible that the three k's are all alike, but we have not enough data to determine their values. The values we adopt for k_1 and k_3 do not, however, affect the values of x'_0 and z'_0, which have been previously given.

PHYSICAL ASPECTS OF THE ASYMMETRY

The motions of the brighter stars have been well explained by Jeans[1] through the gravitational action of the known stars in the local or "Kapteyn" universe. As a general relationship exists between the mean stellar velocity, on the one hand, and the density and the dimensions of the gravitating system, on the other, we can readily see that, in order to account for the existence of stars of very high velocity through gravitational action of the system as a whole, we must include in our study a system of much larger dimensions.

The direction of the asymmetry is nearly in the galactic plane and nearly perpendicular to the direction toward the center of the system of globular clusters as determined by Shapley.[2] This center is in the direction of about 325° galactic longitude, whereas the longitude of the translation of our local system relative to fast-moving stars and distant objects is about 71°. The velocity of translation is about 300 km/sec., and it is possible that the local system moves around the center of the larger system with this velocity. If there exists any appreciable interaction by encounters between stars belonging to the two systems, the ultimate effect would be an increased dispersion perpendicular to the relative motion and a retardation of the systematic motion. As the sum-total of the energy must remain constant, an increase in dis-

[1] *Monthly Notices*, **82**, 122, 1922.

[2] *Mt. Wilson Contr.*, No. 152; *Astrophysical Journal*, **48**, 170, 1918.

persion must correspond to a definite decrease in group-motion until the latter becomes zero.[1] A disintegration of the local system would then occur, the stars escaping through the fan-shaped rear part of the moving system. Among the fast-moving stars we should then expect to find a mixture of field-stars and stars belonging to the local system. The stars of very high velocity can readily be detected by their large proper motion (even for fairly distant objects) combined with very large radial velocities. It is very doubtful, however, whether the number per unit volume is large enough to produce an appreciable asymmetry in the velocity-distribution, without using a time-scale almost inconceivably great.

The interaction between the larger universe and the smaller system may not necessarily be due to encounters between stars. Stars, like the sun, are probably surrounded by dust and emit corpuscles, and this system of fine particles must statistically be at rest with reference to the system from which it has originated. The system of small particles which has originated from the larger system may have an effect upon the internal motions in the smaller system, but the effect would probably be altogether too small to account for the observed phenomenon.

Another possibility is that the connection between the larger and the smaller systems is not of a gravitational or material nature, but represents a velocity-restriction in a world-frame that coincides with the large system of clusters and spirals. The restriction may then be either an effect of the matter in this frame, or an absolute property of the frame itself. It is a remarkable fact that even the most distant clusters and spirals have velocities that are small compared with that of light. The small velocities are ordinarily supposed to be due to initial conditions, in general the assumption being made that the relative initial velocities were small or zero, or that an exchange of kinetic energy has taken place and produced a distribution-law of velocities relative to the system as a whole.

[1] It is interesting to note that the mean square velocity, $a^2+b^2+c^2$, for a star in group X is of the same order of magnitude as the mean square velocity of the first groups in Table I if their motion is referred to the centroid of group X. The ratio of the squares is 55:71. The mean square velocity of a star belonging to any of the groups I to XI can thus be made nearly the same if referred to the larger frame. The mean kinetic energy varies considerably, however.

But we are far from sure that all stars, clusters, and nebulae have some time or other been more intimately connected than they are now. The fact that their velocities are limited, and that the velocity system is practically the same for objects several hundred thousand light-years apart, may be an effect of a velocity-restriction in a frame that can be statistically determined by their present velocities. The question whether it is a property of gravitating matter in space, of space itself, or of the ether is probably intimately connected with the similar question concerning the origin of inertia. In Einstein's elliptical space-time we have a fundamental system of space-co-ordinates in which all the matter in the universe is permanently at rest in a statistical sense. The result of the present study may indicate that it is relative to this world-frame that the velocity-restriction occurs.

These hypotheses as to the physical significance of the asymmetry are suggested only to arouse discussion about the remarkable distribution of cosmic velocities. In both it is supposed that the individual stars are connected both with the local system and with a world-frame. The nature of the first of these connections must almost of necessity be one of origin, combined with the effect of encounters, especially since we have found that the symmetry in the velocity-distribution can be restored by the introduction of the second connection. The nature of the second connection is very uncertain. The principal result of this study is that we have found reason to believe that an additional connection exists, and that this connection is with a cosmical system of enormous dimensions.

MOUNT WILSON OBSERVATORY
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INTERFERENCE BANDS PRODUCED BY MICA AND THE USE OF MICA WINDOWS IN INFRA-RED SPECTROSCOPY

By CHARLES F. MEYER and DETLEV W. BRONK

ABSTRACT

The effect of interference upon the intensity of infra-red radiation transmitted through mica.—Sheets of mica placed in a beam of infra-red radiation may introduce large variations in intensity in the spectrum, due to interference, as is shown by an illustrative graph. Other graphs show that the variation introduced by a single sheet of mica is regular, and amounts to about 10 per cent in the region of 3.5 μ. When mica is used for windows of *absorption cells*, the use of compensating windows cut from the same sheets of mica is in general not effective in compensating for the variations in intensity due to interference. If a sheet of mica 0.05 mm thick is placed in the beam, with the normal to the sheet making an angle of 30° with the axis of the beam, then the variation in intensity is obliterated.

An interference-free cell.—A cell which eliminates interference effects by the foregoing method and embodies other unique features is described.

It is a familiar laboratory experiment to cause light from an incandescent source to be reflected by a thin sheet of mica, and then to pass into an ordinary spectroscope. When this is done the observed spectrum consists of the continuous spectrum of the source, except that it is crossed at regular intervals by vertical dark bands. The dark bands originate from destructive interference between light reflected from the front and rear surfaces of the sheet of mica.

If the light transmitted by the mica is considered, then it is to be expected that there should be corresponding variations of intensity in passing along the spectrum. Some years ago during an investigation in the infra-red, in which mica windows were being used, variations in the intensity of the spectrum were found. These were ascribed to interference bands produced by the mica. Corrections for them were applied but no specific study of the interference was made.

In some recent work large variations in intensity were found. Mica was, as before, being used for windows of absorption cells, and mica is often the only available material for such windows. The plotting of the variations and the correcting for them is labori-

ous, and they may be so large that overlooking them would lead into great error. The nature and magnitude of the variations obtained in this recent investigation are shown by the graph in Figure 1. The range of variation is over 30 per cent of the normal intensity.

A study of the interference was undertaken, and a method has been found of overcoming the variations in intensity. The method is applicable for a convergent or divergent beam, but not for one that is parallel or nearly so. But this is no serious limitation, as it is not usual to have a beam parallel where it passes through an absorption chamber.

In studying the interference, a single sheet of clear mica, 0.05 mm thick, was used. The sheet was examined under crossed nichols. From the regions of its surface where a change in shade of the transmitted light indicated a change in thickness, a very thin overlying layer was scaled off, until further examination showed no change in shade. The sheet was thus uniform in thickness. Other methods of examining sheets of mica were also tried, but the method of crossed nichols seemed the best. In addition the sheets were also examined by viewing them against a dark background, and reflecting light

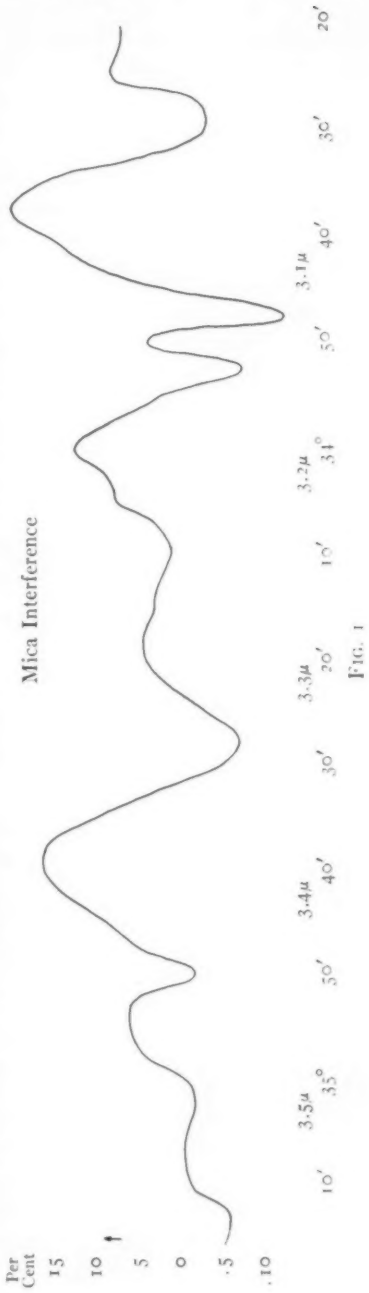

FIG. 1

from them. A sheet showing separation along a cleavage plane must be rejected.

The sheet was mounted normal to the axis of the beam, in such a manner that it could be readily moved into the beam or out of it. In Figure 2, curve *A*, is shown the variation in intensity (ordinates), against circle settings of the spectrometer (abscissae). Differences in these are very nearly proportional to wave-length differences, so that a scale of wave-lengths has also been put on the chart. From the graph it appears that the rise and fall of intensity for a single

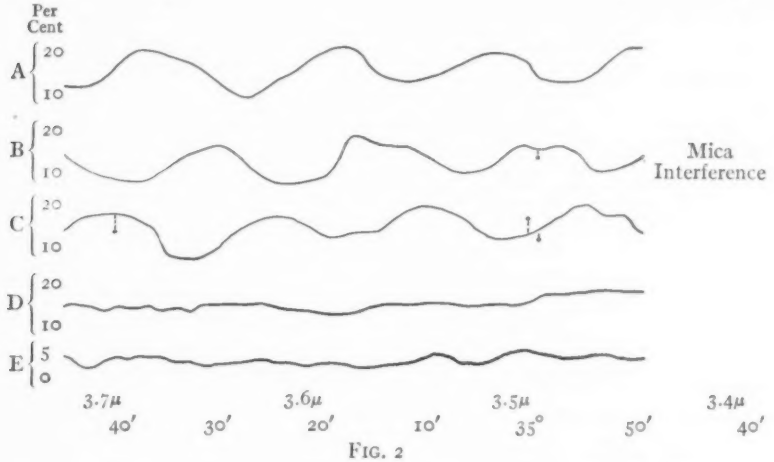

FIG. 2

sheet of mica is regular, as might be expected. The small irregularities are to be ascribed to accidental disturbances of the galvanometer. The variation of intensity is seen to be about 10 per cent.

If the normal to the sheet is inclined at a small angle to the beam, the maxima and minima of intensity should be shifted, for the thickness of mica traversed would be slightly increased. In curve *B* is shown the variation of intensity for an angle of 10° between the normal to the sheet and the beam. The shift with reference to curve *A* is apparent.

On the other hand, if the radiation were allowed to pass through a different portion of the uniform sheet, and the sheet were again placed normal to the beam, then one would expect to find the same curve as curve *A*. But on trying this, curve *C* was obtained, which, it will be noted, is also shifted with reference to curve *A*. This has

an important bearing upon the use of mica for cell windows. It is ordinarily desirable to have two windows for the cell proper, and two compensating windows which are cut from the same sheet of mica. The comparison of curves *A* and *C* shows that even when great precaution is taken to have uniform sheets of mica, and when ordinarily practicable precautions are taken to keep both of the surfaces normal to the beam, the variations due to interference cannot be relied on to compensate for each other. And under the most unfavorable conditions, due to the four windows, the variations might amount to nearly 40 per cent.

It is instructive to apply the theory of interference in parallel plates quantitatively to the present experiment. The advance of the disturbance directly transmitted over the disturbance which is twice internally reflected is $\frac{2\mu e}{\lambda}\cos\theta$ wave-lengths, where μ is the refractive index, e the thickness of the plate, λ the wave-length of the radiation, and θ the angle between the normal to the plate and the direction of the ray within the plate. Taking μ as 1.5, e as 0.05 mm, and $\lambda = 0.0033$ mm, we have $\frac{2\mu e}{\lambda} = 45$, approximately. For normal incidence there is a retardation of 45 wave-lengths.

Due to the convergence of the beam all of the rays cannot be incident normally. The beam has an angle of about 10°. If the central ray is normal, the extreme rays will be incident at an angle of 5°. They will be refracted at an angle of

$$\sin^{-1}\left(\frac{\sin 5^\circ}{1.5}\right) = 3^\circ\,20'.$$

Now $\cos 3^\circ 20' = 0.9983$. The variation in retardation taken over the beam is then $45\times(1-0.9983) = 45\times 0.0017 = 0.076$ wave-length. In a similar manner, if the normal to the sheet makes an angle of 10° with the central ray, then the variation of retardation over the beam is

$$45\left[\cos\sin^{-1}\left(\frac{\sin 5^\circ}{1.5}\right) - \cos\sin^{-1}\left(\frac{\sin 15^\circ}{1.5}\right)\right].$$

The value of the first term in the brackets we have above; the second term,

$$\cos \sin^{-1}\left(\frac{\sin 15^\circ}{1.5}\right) = \cos 9^\circ 58' = 0.9849.$$

Therefore, the variation of retardation is $45\ (0.9983 - 0.9849) = 0.6$ wave-length. Some obliteration of the variation of intensity due to the interference might be expected under these conditions, but comparison of curves *A* and *B* does not give any evidence thereof. It must be remembered, however, that accidental errors prevent close comparison to the two curves. The accidental errors may amount to 2 or 3 per cent.

When the sheet is inclined so as to make an angle of 30° with the central ray, a similar calculation shows that the variation of retardation over the beam is 1.6 wave-lengths. Curve *D* shows the variation of intensity along the spectrum. Apparently it has been obliterated. The small irregularities in the curve are probably due to accidental error.

A cell with four windows was then tried, the windows making an angle of 30° with the axis of the beam. Curve *E* shows the variation of intensity. When this curve is compared with the curve shown in Figure 1, the advantage of having the mica windows at an angle, to suppress the variation of intensity, is obvious.

By way of conclusion it may be of interest to describe an absorption cell which is now in use. This cell was designed with a view to embodying the principle that the normal to the cell windows should make an angle with the beam; and the cell is of variable length. It is represented in Figure 3. Two pieces of standard-size brass tubing, about 10 cm in diameter and 21 cm long, were chosen. The diameter of one piece is larger than the other by enough so that the larger tube *A* will just slide over the smaller tube *B* (Fig. 3.). One end of each piece is cut off at 90° to the axis (square), the other at 60° to the axis. An oval flange 12 mm wide is soldered over the end which is cut at 60°. The flange is flush with the outside of each tube, and projects over the inside (*E* and *F*, Fig. 3). A mica window is sealed to each flange with wax. By telescoping the tubes in such a manner that the

windows are at the same end (as shown in the figure), the cell can be made as short as desired. And by telescoping with the windows at opposite ends, the cell can be made long, the length

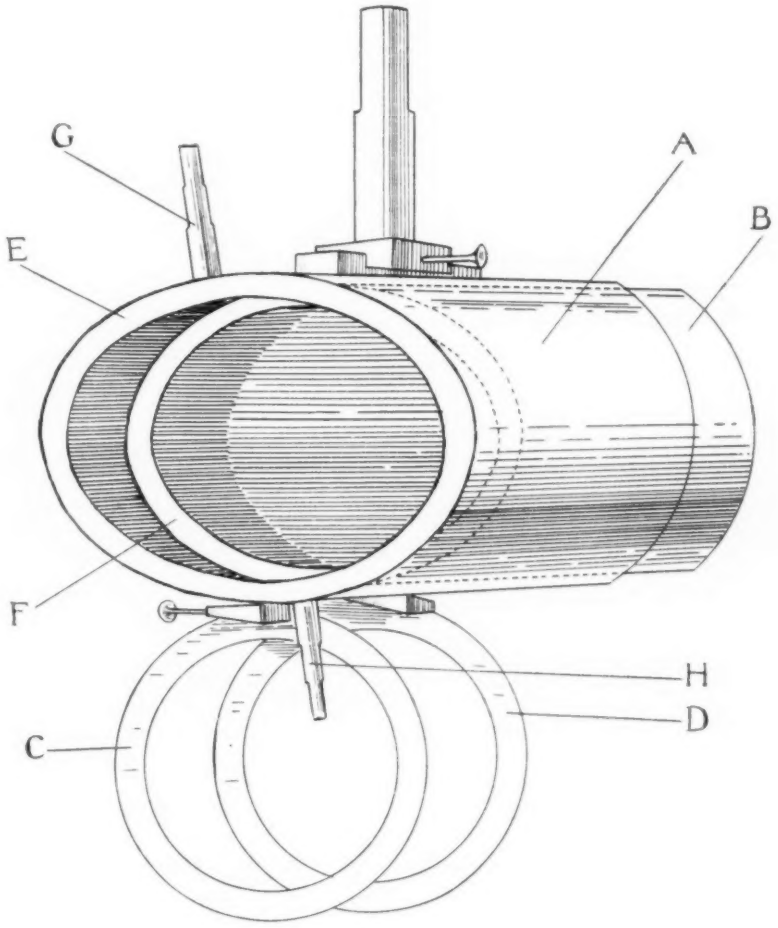

FIG. 3

being again variable, of course. For some intermediate lengths it is necessary to have a short piece of the tubing of smaller diameter which is cut off square at both ends. This has a flange soldered completely inside of it, at an angle of 60° also.

Each window for the cell proper is cut from a uniform sheet of mica large enough to yield another window, used for compensating, when the cell is taken out of the beam. The compensating windows are mounted below the cell in such a manner that when they in turn are in the beam, they are parallel to the positions which the cell windows occupy when in the beam. The mountings for the compensating windows are represented by *C* and *D*.

There are three side tubes of 5 mm diameter, provided for inlet and outlet of the gas to be studied. Two of these, *G* and *H*, are very near the end of *A*. They are at opposite ends of a diameter, and so near the mica window that they still serve, even when the cell is but a millimeter long. The third side tube is on *B*. It is removable, and is not shown in the figure. It serves when the cell tubes *A* and *B* are telescoped in the manner to yield a long cell. It is then desirable to have the inlet at one end, and the outlet at the other, in order to facilitate filling the cell.

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Reviews

X-Rays. By G. W. C. Kaye. Fourth Edition. London: Longmans, Green & Co., 1923. Pp. xxii+320. 125 illustrations.

The first three editions of this book have found such universal use among workers on this subject that there is little to be said in a review concerning those parts that are not devoted to new material. No other text in the English language contains so much material of service to the experimenter, or to the man who wishes to make an application of X-rays to practical uses. On the other hand it does not pretend to enter thoroughly into a discussion of theoretical problems, merely giving very brief summaries of such, and bibliographical references to the original papers or to other more complete analyses.

The first important change from former editions is found in chapter xi on "Practical Applications of X-Rays," where the latter part of the chapter dealing with the application to medicine and to industry has been rewritten and enlarged. The various examples given here of tests for defective materials and for frauds would make extremely interesting reading, even to the man who knows almost nothing of the subject. To the student they reveal many valuable possibilities of development.

The last part of chapter xiii on "Diffraction of X-Rays by Crystals" has, quite naturally, been more thoroughly revised and enlarged than any other part of the book. Experimental work in this part of the field is advancing so rapidly that it is of course impossible for any text to be even approximately complete.

A sixth appendix has been added giving a glossary of X-ray and electro-medical nomenclature. Many authors on other subjects writing for students and for others who are not specialists might do well to imitate this feature.

The amount of revision in this edition is hardly enough to warrant its purchase by the owner of a copy of the preceding edition.

Dinsmore Alter

ERNEST FOX NICHOLS

It is with great regret and with a deep sense of personal loss that we record the death, on April 29, 1924, of Ernest Fox Nichols, who has been a collaborator of this *Journal* since 1898. Mr. Nichols had nearly concluded the presentation of a paper at the initial scientific session of the National Academy of Sciences in its splendid new building in Washington, when he paused, sank into a seat, and passed away instantly. He had been in ill health for the past three years, and such a fatality was not wholly unexpected by his family and friends.

The paper, entitled "Joining the Infra-red and Electric Wave Spectra," described the detection by Mr. Nichols, in collaboration with Mr. J. D. Tear, of electric radiations of wave-lengths 0.3 mm to 10 mm.

His service to science and to education had been as professor of physics, successively, at Colgate University (1892–98), Dartmouth College (1898–1903), Columbia University (1903–9), Yale University (1916–20); as president of Dartmouth College (1909–16), and as director of pure science at the Research Laboratory of the National Electric Lamp Association, Cleveland, since 1920. He was inaugurated as president of the Massachusetts Institute of Technology in June, 1921, but was unable to undertake the duties of that position on account of his ill health.

We shall publish in a future number a biographical sketch, and the paper the presentation of which so dramatically closed his scientific career in the fifty-fifth year of his life.